MIT Joint Program on the Science and Policy of Global Change

Computable General Equilibrium Models and Their Use in Economy-Wide Policy Analysis

Ian Sue Wing

Technical Note No. 6
September 2004

The MIT Joint Program on the Science and Policy of Global Change is an organization for research, independent policy analysis, and public education in global environmental change. It seeks to provide leadership in understanding scientific, economic, and ecological aspects of this difficult issue, and combining them into policy assessments that serve the needs of ongoing national and international discussions. To this end, the Program brings together an interdisciplinary group from two established research centers at MIT: the Center for Global Change Science (CGCS) and the Center for Energy and Environmental Policy Research (CEEPR). These two centers bridge many key areas of the needed intellectual work, and additional essential areas are covered by other MIT departments, by collaboration with the Ecosystems Center of the Marine Biology Laboratory (MBL) at Woods Hole, and by short- and long-term visitors to the Program. The Program involves sponsorship and active participation by industry, government, and non-profit organizations.

To inform processes of policy development and implementation, climate change research needs to focus on improving the prediction of those variables that are most relevant to economic, social, and environmental effects. In turn, the greenhouse gas and atmospheric aerosol assumptions underlying climate analysis need to be related to the economic, technological, and political forces that drive emissions, and to the results of international agreements and mitigation. Further, assessments of possible societal and ecosystem impacts, and analysis of mitigation strategies, need to be based on realistic evaluation of the uncertainties of climate science.

The Program's Technical Notes series is intended to document technical research details for our own use, and also for other researchers who may wish to recreate or update the work. Further information is available by request to the Program Office at the numbers listed below.

Henry D. Jacoby and Ronald G. Prinn,
Program Co-Directors

For more information, please contact the Joint Program Office

Postal Address: Joint Program on the Science and Policy of Global Change
MIT E40-428
77 Massachusetts Avenue
Cambridge MA 02139-4307 (USA)

Location: One Amherst Street, Cambridge
Building E40, Room 428
Massachusetts Institute of Technology

Access: Phone: (617) 253-7492
Fax: (617) 253-9845
E-mail: globalchange@mit.edu
Web site: http://mit.edu/globalchange/

Printed on recycled paper

Computable General Equilibrium Models and Their Use in Economy-Wide Policy Analysis: Everything You Ever Wanted to Know (But Were Afraid to Ask)

Ian Sue Wing*

Center for Energy & Environmental Studies and Department of Geography & Environment
Boston University
and
Joint Program on the Science & Policy of Global Change
Massachusetts Institute of Technology

* Address: Rm. 141, 675 Commonwealth Ave., Boston MA 02215. Email: isw@bu.edu. Phone: (617) 353-5741. Fax: (617) 353-5986.

This research was supported by U.S. Department of Energy Office of Science (BER) Grant No. DE-FG02-02ER63484.

Computable General Equilibrium Models
and Their Use in Economy-Wide Policy Analysis:
Everything You Ever Wanted to Know (But Were Afraid to Ask)

Abstract

This paper is a simple, rigorous, practically-oriented exposition of computable general equilibrium (CGE) modeling. The general algebraic framework of a CGE model is developed from microeconomic fundamentals, and employed to illustrate (i) how a model may be calibrated using the economic data in a social accounting matrix, (ii) how the resulting system of numerical equations may be solved for the equilibrium values of economic variables, and (iii) how perturbing this equilibrium by introducing tax or subsidy distortions facilitates analysis of policies' economy-wide impacts.

JEL Classification: C68, D58, H22, Q43
Keywords : general equilibrium, CGE models, policy analysis, taxation, climate change policy

1. Introduction

Walrasian general equilibrium prevails when supply and demand are equalized across all of the interconnected markets in the economy. Computable general equilibrium (CGE) models are simulations that combine the abstract general equilibrium structure formalized by Arrow and Debreu with realistic economic data to solve numerically for the levels of supply, demand and price that support equilibrium across a specified set of markets. CGE models are a standard tool of empirical analysis, and are widely used to analyze the aggregate welfare and distributional impacts of policies whose effects may be transmitted through multiple markets, or contain menus of different tax, subsidy, quota or transfer instruments. Examples of their use may be found in areas as diverse as fiscal reform and development planning (e.g., Perry et al 2001; Gunning and Keyzer 1995), international trade (e.g., Shields and Francois 1994; Martin and Winters 1996; Harrison et al 1997), and increasingly, environmental regulation (e.g., Weyant 1999; Bovenberg and Goulder 1996; Goulder 2002).

CGE models' usefulness notwithstanding, they are nonetheless viewed with suspicion by some in the economics and policy analysis communities as a "black box", whose results cannot be meaningfully traced to any particular features of their data base or input parameters, algebraic structure, or method of solution. Such criticism typically rests on the presumptions that CGE models contain a large number of variables and parameters and are structurally complex, both of which allow questionable assumptions to be hidden within them that end up driving their results. A good example is Panagariya and Duttagupta (2001), who note in the context of trade liberalization that:

> "Unearthing the features of CGE models that drive [their results] is often a time-consuming exercise. This is because their sheer size, facilitated by recent advances in computer technology, makes it difficult to pinpoint the precise source of a particular result. They often remain a black box. Indeed, frequently, authors are themselves unable to explain their results intuitively and, when pressed, resort to uninformative answers..."

While this view contains a kernel of truth, it is also symptomatic of misunderstanding of the simplicity of the algebraic foundation at the core of all CGE models—regardless of their size or apparent complexity, the key features of the data that these models use, the numerical calibration methods by which models employ these data to imbue their algebraic framework with empirical substance, and the operations research techniques by which the resulting mathematical programming problem is solved to generate the results that are then quoted in the policy literature.

The problem is that much of this information is often not communicated in manner that is accessible to the broader economics or policy communities. Descriptions of models' underlying structure, calibration and solution methods abound, but tend to be spread across a broad cross-section of materials, each subset of which focuses on a different aspect of the subject. Of the numerous articles that use CGE simulations, the majority document only those attributes of their models that are relevant to the application at hand (e.g., Jacoby and Sue Wing 1998), or merely present the model's equations with little explanation to accompany them (e.g., Bovenberg and Goulder 1996). Expositions in books and manuals devoted to modeling techniques (e.g., Shoven and Whalley 1992; Ginsburgh and Keyzer 1997; Lofgren et al 2002) tend to be exhaustively detailed, and those in articles focused on applied numerical optimization (e.g., Rutherford 1995; Ferris and Pang 1997) often involve a high level of mathematical abstraction, neither of which make it easy for the uninitiated to quickly grasp the basics. Finally, although pedagogic articles (e.g., Devarajan et al 1997; Rutherford 1999; Rutherford and Paltsev 1999; Paltsev 2004) often provide a lucid introduction to the fundamentals, they tend to emphasize either models' structural descriptions or the details of the mathematical software packages used to build them, and have given short shrift to CGE models' theoretical basis or procedures for calibration.

It is therefore difficult to find an article-length introduction to the subject that integrates these disparate elements into a practical, intuitive explanation of the methods by which CGE models are constructed, calibrated, solved, and used to analyze the impacts of policies with economy-wide effects.[1] This gap in the literature motivates the present paper, whose aim is to de-mystify CGE models by opening up the black box to scrutiny, and to increase their accessibility to a wider group of economists and policy analysts—students, practitioners and academics alike—who would otherwise remain unfamiliar with, and suspicious of, them.

In line with its pedagogical objective, the paper is deliberately simple. In the spirit of Shoven and Whalley (1984), Kehoe and Kehoe (1995), and Kehoe (1998a), it employs the microeconomic foundations of consumer and producer maximization to develop a framework that is not just straightforward but is also sufficiently general to represent a CGE model of arbitrary size and dimension. This framework is then used to demonstrate in a practical fashion how a social accounting matrix may be used to calibrate the coefficients of the model equations, how the resulting system of numerical equations is solved, and how the equilibrium thus solved for may be perturbed and the results used to analyze the economic effects of policies.

To specialists who are already familiar with CGE models, there will be little in this paper that is new, as the aforementioned techniques of model formulation, specification, calibration and solution are all well established. The primary contribution of the paper is the framework that it develops to integrate these elements in a transparent and step-by-step manner, creating a pedagogic digest that can serve as an introduction to the subject of CGE modeling that is simple and practically oriented, yet also theoretically coherent and reasonably comprehensive. The hope is that this will not only alleviate some of the general suspicion about CGE models, but will also facilitate and promote their use as a tool for policy analysis by giving the reader an appreciation of their simplicity and power.

[1] But see recent exceptions by Kehoe (1998a) and Boehringer et al (2003).

The plan of the paper is as follows. Section 2 introduces the circular flow of the economy, and demonstrates how it serves as the fundamental conceptual starting point for Walrasian equilibrium theory that underlies a CGE model. Section 3 presents a social accounting matrix and shows how the algebra of its accounting rules reflects the conditions of general equilibrium. Section 4 develops these relationships into a CGE model using the device of the Cobb-Douglas (C-D) economy in which households have C-D preferences and firms have C-D production technology. Section 5 discusses techniques of model formulation, solution and numerical calibration. Section 6 explains the use of CGE models to analyze the incidence and welfare effects of taxes, and section 7 provides a practical demonstration using a stylized numerical example. A more realistic example is presented in section 8, which applies a CGE simulation of the C-D economy to U.S. data for the purpose of elucidating the general equilibrium effects of taxing carbon dioxide emissions to mitigate global warming. Section 9 provides a summary and conclusion.

2. Foundations: The Circular Flow and Walrasian Equilibrium

The fundamental conceptual starting point for a CGE model is the circular flow of commodities in a closed economy, shown in Figure 1.[2] The main actors in the diagram are households, who own the factors of production and are the final consumers of produced commodities, and firms, who rent the factors of production from the households for the purpose of producing goods and services that the households then consume. Many CGE models also explicitly represent the government, but its role in the circular flow is often passive: to collect taxes and disburse these revenues to firms and households as subsidies and lump-sum transfers, subject to rules of budgetary balance that are specified by the analyst. In tracing the circular flow one can start with the supply of factor inputs (e.g. labor and capital services) to the firms and continue to the supply of goods and services from the firms to the households, who in turn control the supply of factor services. One may also begin with payments, which households receive for the services of labor and capital provided to firms by their primary factor endowment, and which are then used as income to pay producing sectors for the goods and services that the households consume.

Equilibrium in the economic flows in Figure 1 results in the conservation of both product and value. Conservation of product, which holds even when the economy is not in equilibrium, reflects the physical principle of material balance that the quantity of a factor with which households are endowed, or of a commodity that is produced by firms, must be completely absorbed by the firms or households (respectively) in the rest of the economy. Conservation of value reflects the accounting principle of budgetary balance that for each activity in the economy the value of expenditures must be balanced by the value of incomes, and that each unit of expenditure has to purchase *some* amount of some type of commodity. The implication is that neither product nor value can appear out of nowhere: each activity's production or endowment must be matched by others' uses, and each activity's income must be balanced by others' expenditures. Nor can product or value disappear: a transfer of purchasing power can only be effected through an opposing transfer of some positive amount of some produced good or primary factor service, and vice versa.

These accounting rules are the cornerstones of Walrasian general equilibrium. Conservation of product, by ensuring that the flows of goods and factors must be absorbed by the production and consumption activities in the economy, is an expression of the principle of no

[2] This discussion is adapted from Babiker et al (2001).

free disposability. It implies that firms' outputs are fully consumed by households, and that households' endowment of primary factors is in turn fully employed by firms. Thus, for a given commodity the quantity produced must equal the sum of the quantities of that are demanded by the other firms and households in the economy. Analogously, for a given factor the quantities demanded by firms must completely exhaust the aggregate supply endowed to the households. This is the familiar condition of *market clearance*.

Conservation of value implies that the sum total of revenue from the production of goods must be allocated either to households as receipts for primary factors rentals, to other industries as payments for intermediate inputs, or to the government as taxes. The value of a unit of each commodity in the economy must then equal the sum of the values of all the inputs used to produce it: the cost of the inputs of intermediate materials as well as the payments to the primary factors employed in its production. The principle of conservation of value thus simultaneously reflects constancy of returns to scale in production and perfectly competitive markets for produced commodities. These conditions imply that in equilibrium producers make *zero profit*.[3]

Lastly, the returns to households' endowments of primary factors, that are associated with the value of factor rentals to producers, accrue to households as income that the households exhaust on goods purchases. The fact that households' factor endowments are fully employed, so that no amount of any factor is left idle, and that households exhaust their income on commodity purchases (some amount of which are for the purpose of saving), reflects the principle of balanced-budget accounting known as *income balance*. One can also think of this principle as a zero profit condition on the production of a "utility good", whose value is the aggregate of the values of households' expenditures on commodities, and whose price is the marginal utility of income.

The three conditions of market clearance, zero profit and income balance are employed by CGE models to solve simultaneously for the set of prices and the allocation of goods and factors that support general equilibrium. The three conditions define Walrasian general equilibrium not by the *process* of exchange by which this allocation comes about, but in terms of the allocation itself, which is made up of the components of the circular flow shown by solid lines in Figure 1. General equilibrium can therefore be modeled in terms of barter trade in commodities and factors, without the need to explicitly keep track of—or even represent—the compensating financial transfers. Consequently, CGE models typically do not explicitly represent money as a commodity. However, in order to account for such trades the quantities of different commodities still need to be made comparable by denominating their values in some common unit of account. The flows are thus expressed in terms of the value of one commodity—the so-called numeraire good—whose price is taken as fixed. For this reason, CGE models only solve for relative prices. This point is elaborated later on in Section 4.

3. The Algebra of Equilibrium and the Social Accounting Matrix

The implications of the circular flow for both the structure of CGE models and the economic data on which they are calibrated are clearly illustrated in an algebraic framework. To this end, consider a hypothetical closed free-market economy that is composed of N industries, each of which produces its own type of commodity, and an unspecified number of households that jointly own an endowment of F different types of primary factors.

[3] Together, these conditions imply that with unfettered competition firms will continue to enter the economy's markets for goods until profits are competed away to zero.

To keep things simple we make three assumptions about this economy. First, there are no tax or subsidy distortions, or quantitative restrictions on trade. Second, the households act collectively as a single representative agent who rents out the factors to the industries in exchange for income. Households then spend the latter to purchase the *N* commodities for the purpose of satisfying *D* types of demands (e.g., demands for goods for the purposes of consumption and investment). Third, each industry behaves as a representative firm that hires inputs of the *F* primary factors and uses quantities of the *N* commodities as intermediate inputs to produce a quantity *y* of its own type of output.

Then, letting the indices $i = \{1, ..., N\}$ denote the set of commodities, $j = \{1, ..., N\}$ the set of industry sectors, $f = \{1, ..., F\}$ the set of primary factors, and $d = \{1, ..., D\}$ the set of final demands, the circular flow in this economy can be completely characterized by three data matrices: an $N \times N$ input-output matrix of industries' uses of commodities as intermediate inputs, denoted by $\overline{\mathbf{X}}$, an $F \times N$ matrix of primary factor inputs to industries, denoted by $\overline{\mathbf{V}}$, and an $N \times D$ matrix of commodity uses by final demand activities, denoted by $\overline{\mathbf{G}}$.

It is straightforward to establish how the elements of the three matrices may be arranged to reflect the logic of the circular flow. First, commodity market clearance implies that the value of gross output of industry *i*, which is the value of the aggregate supply of the i^{th} commodity, $\overline{y}_i$, must equal the sum of the values of the *j* intermediate uses of that good, $\overline{x}_{ij}$, and the *d* final demands $\overline{g}_{id}$ that absorb that commodity:

(1) $$\overline{y}_i = \sum_{j=1}^{N} \overline{x}_{ij} + \sum_{d=1}^{D} \overline{g}_{id} .$$

Similarly, factor market clearance implies that the firms in the economy fully employ the representative agent's endowment of a particular factor, $\overline{V}_f$:

(2) $$\overline{V}_f = \sum_{i=1}^{N} \overline{v}_{fj} .$$

Second, the fact that industries make zero profit implies that the value of gross output of the j^{th} sector, $\overline{y}_j$, must equal the sum of the benchmark values of inputs of the *i* intermediate goods $\overline{x}_{ij}$ and *f* primary factors $\overline{v}_{fj}$ that the industry employs in its production:

(3) $$\overline{y}_j = \sum_{i=1}^{N} \overline{x}_{ij} + \sum_{i=1}^{F} \overline{v}_{fj} .$$

Third, the representative agent's income, $\overline{m}$, is made up of the receipts from the rental of primary factors—none of which remain idle, and must balance the agent's gross expenditure on satisfaction of commodity demands. Together, these conditions imply that income must equal the sum of the elements of $\overline{\mathbf{V}}$, which in turn must equal the sum of the elements of $\overline{\mathbf{G}}$. Thus, by (2),

(4) $$\overline{m} = \sum_{f=1}^{F} \overline{V}_f = \sum_{i=1}^{N} \sum_{d=1}^{D} \overline{g}_{id} .$$

The accounting relationships in eqs. (1)-(4) jointly imply that, in order to reflect the logic of the circular flow, the matrices $\overline{\mathbf{X}}$, $\overline{\mathbf{V}}$ and $\overline{\mathbf{G}}$ should be arranged according to Figure 2(a). This diagram is an accounting tableau known as a social accounting matrix (SAM), which is a snapshot of the inter-industry and inter-activity flows of value within an economy that is in equilibrium in a particular benchmark period. The SAM is an array of input-output accounts that are denominated in the units of value of the period for which the flows in the economy are recorded, typically the currency of the benchmark year. Each account is represented by a row and a column, and the cell elements record the payment from the account of a column to the account of a row. Thus, an account's components of income of (i.e., the value of receipts from the sale of a commodity) appear along its row, and the components of its expenditure (i.e., the values of the inputs to a demand activity or the production of a good) appear along its column (King 1985).

The structure the SAM reflects the principle of double-entry book-keeping, which requires that for each account, total revenue—the row total—must equal total expenditure—the column total. This is apparent from Figure 2(a), where the sum across any row in the upper quadrants $\overline{\mathbf{X}}$ and $\overline{\mathbf{G}}$ is equivalent to the expression for goods market clearance from eq. (1), and the sum across any row in the south-west quadrant $\overline{\mathbf{V}}$ is equivalent to the expressions for factor market clearance from eq. (2). Likewise, the sum down any column of the left-hand quadrants $\overline{\mathbf{X}}$ and $\overline{\mathbf{V}}$ is equivalent to the expression for zero-profit in industries from eq. (3). Furthermore, once these conditions hold, the sums of the elements of the northeast and southwest quadrants ($\overline{\mathbf{G}}$ and $\overline{\mathbf{V}}$, respectively) should equal one another, which is equivalent to the income balance relationship from eq. (4) that reflects the intuition that GDP (the aggregate of the components of expenditure) is equal to value added (the aggregate of the components of income). The fact that these properties are the expression of Walrasian general equilibrium makes the SAM an ideal data base from which to construct a CGE model.

4. From a SAM to a CGE Model: The Cobb-Douglas Economy

CGE models' algebraic framework results from the imposition of the axioms of producer and consumer maximization on the accounting framework of the SAM. To illustrate this point we use the pedagogic device of a "Cobb-Douglas economy" in which households are modeled as a representative agent that is assumed to have C-D preferences and industry sectors are modeled as representative producers that are assumed to have C-D production technologies. In the former case, household utility, U, is related to the consumption c of the N commodities by the C-D function:

$$(5)\qquad U = A_C c_1^{\alpha_1} c_2^{\alpha_2} \ldots c_N^{\alpha_N} = A_C \prod_{i=1}^{N} c_i^{\alpha_i} ,$$

where the exponent parameters α_i are the shares of each good in expenditure on consumption (so that $\alpha_1 + \ldots + \alpha_N = 1$), and A_c is a scaling parameter. In the latter case, the output of the j^{th} industry, y_j, is a recipe for combining inputs of the N types of intermediate goods, x, and the F varieties of primary factors v according to the C-D function:

$$(6)\qquad y_j = A_j \left(x_1^{\beta_1} x_2^{\beta_2} \ldots x_N^{\beta_N}\right)\left(v_1^{\gamma_1} v_2^{\gamma_2} \ldots v_F^{\gamma_N}\right) = A_j \prod_{i=1}^{N} x_{ij}^{\beta_{ij}} \prod_{f=1}^{F} v_{fj}^{\gamma_{fj}} ,$$

where the exponent parameters β_i and γ_i denote the shares of each input in the cost of production (so that $\beta_{1j} + \ldots + \beta_{Nj} + \gamma_{1j} + \ldots + \gamma_{Nj} = 1$), and A_j is a scaling parameter.[4]

4.1. *Households*

The treatment of households mirrors that in the previous section. Assume a representative agent that maximizes utility by choosing the levels of consumption of the N commodities in the

[4] The use of the C-D function is ubiquitous in economics. Its origins are described in Samuelson (1979) and Sandelin (1976). Humphrey (1997) provides a fascinating account of earlier algebraic representations of production.

economy, subject to the constraints of her income, m, ruling commodity prices, p. The agent may also demand goods and services for purposes other than consumption—in the present example saving s—which are assumed to be exogenous and constant. The agent's problem is thus:

$$(7) \qquad \max_{c_i} U(c_1, \ldots, c_N) \qquad \text{subject to}$$

$$m = \sum_{i=1}^{N} p_i (c_i + s_i).$$

We assume that the representative agent has C-D preferences, so that $U(\cdot)$ is specified according to eq. (5) above. It is equivalent but advantageous to re-formulate this problem as one of household production, in which the representative agent maximizes the "profit" from the production of a "utility good" U whose output is generated by consumption, and whose price p_U is the marginal utility of aggregate consumption, which can be treated as the numeraire price in the economy. Thus, eq. (7) is equivalent to the problem:

$$(8) \qquad \max_{c_i} p_U U - \sum_{i=1}^{N} p_i c_i$$

subject to the definition of utility above. Solving this problem yields the representative agent's demand function for the consumption of the i^{th} commodity:[5]

$$(9) \qquad c_i = \alpha_i \frac{\left(m - \sum_{i=1}^{N} p_i s_i \right)}{p_i}.$$

Rearranging this expression yields $\alpha_i = \frac{c_i p_i}{\left(m - \sum_{i=1}^{N} p_i s_i \right)}$, which indicates that the exponents of the utility function may be interpreted as the shares of each commodity in the total value of consumption. Other components of final demand (e.g., saving or investment) may be easily handled as sinks for product that are directly specified as demand functions, or the agent's utility function may be extended to incorporate the representative agent's preferences for other categories of expenditure.

4.2. Producers

Each producer maximizes profit π by choosing levels of intermediate inputs and primary factors to produce output, subject to the constraint of its production technology ϕ. The j^{th} producer's problem is thus:

$$(10) \qquad \max_{x_{ij}, v_{fj}} \pi_j = p_j y_j - \sum_{i=1}^{N} p_i x_{ij} - \sum_{f=1}^{F} w_f v_{fj} \quad \text{subject to}$$

$$y_j = \phi_j \left(x_{1j}, \ldots, x_{Nj}; v_{1j}, \ldots, v_{Fj} \right).$$

Let each producer have C-D production technology, so that $\phi(\cdot)$ is specified according to eq. (6) above. Solving the problem in (10) yields producer j's demands for intermediate inputs of commodities:[6]

[5] The details are given in Appendix A.
[6] The details are given in Appendix B.

(11) $$x_{ij} = \beta_{ij} \frac{p_j y_j}{p_i},$$

and its demands for primary factor inputs:

(12) $$v_{fj} = \gamma_{fj} \frac{p_j y_j}{w_f}.$$

Rearranging eqs. (11) and (12) yields $\beta_{ij} = \frac{p_i x_{ij}}{p_j y_j}$ and $\gamma_{fj} = \frac{w_f v_{fj}}{p_j y_j}$, respectively, which show that, similar to the demand for consumption goods above, the exponents of the Cobb-Douglas production function represent the shares of their respective inputs to production in the value of output.

4.3. General Equilibrium

Eqs. (9), (11) and (12) are the building blocks from which a CGE model is constructed. What bind these elements together are the general equilibrium conditions that are developed algebraically in section 3, which must be re-formulated for the Cobb-Douglas economy. Once these conditions are properly specified, (9), (11) and (12) may be substituted into them to yield the actual equations that a CGE model uses to solve for equilibrium.

In the C-D economy the conditions for general equilibrium are as follows. Market clearance implies that the quantity of each commodity produced must equal the sum of the quantities of that commodity demanded by the *j* producers in the economy as an intermediate input to production, and by the representative agent as an input to consumption and saving activities. Thus, eq. (1) becomes:

(13) $$y_i = \sum_{j=1}^{N} x_{ij} + c_i + s_i.$$

In addition, the quantities of primary factor *f* used by all producers must sum to the representative agent's endowment of that factor, V_f. From eq. (2) this condition implies:

(14) $$V_f = \sum_{j=1}^{N} v_{fj}.$$

Zero profit implies that the value of output generated by producer *j* must equal the sum of the values of the inputs of the *i* intermediate goods and *f* primary factors employed in production. This condition is easily deduced by setting the right-hand side of eq. (10) to zero and rearranging:

(15) $$p_j y_j = \sum_{i=1}^{N} p_i x_{ij} - \sum_{f=1}^{F} w_f v_{fj},$$

which is the analogue of is eq. (3). Income balance implies that the income of the representative agent must equal the value of producers' payments to her for the use of the primary factors that she owns and hires out. Thus, as in eq. (4):

(16) $$m = \sum_{f=1}^{F} w_f V_f.$$

With these four expressions in hand the equations that form the core of a CGE model may be easily specified. Assume for simplicity that the endowment of the representative agent is fixed at the instant in time in which general equilibrium prevails. Then, substituting (9) and (11) into eq. (13), and (12) into eq.(14) yields two excess demand vectors that define the divergence Δ^C between supply and demand in the market for each commodity and the divergence Δ^F between supply and demand in the market for each primary factor. The absolute values of both of these sets of differences are minimized to zero in general equilibrium. There are N such excess demand equations for the commodity market:

$$(17) \qquad \Delta_i^C = \sum_{j=1}^{N} \beta_{ij} p_j y_j + \alpha_i \left(\sum_{f=1}^{F} w_f V_f - \sum_{j=1}^{N} p_j s_j \right) + p_i s_i - p_i y_i$$

and F equations for the factor market:

$$(18) \qquad \Delta_f^F = \sum_{j=1}^{N} \gamma_{fj} \frac{p_j y_j}{w_f} - V_f \, .$$

The zero profit condition implies that the absolute value of producers' profits is minimized to zero in general equilibrium. Thus, substituting eqs. (11) and (12) into the production function allows us to write N pseudo-excess demand functions that specify the per-unit excess profit (i.e. excess of price over unit cost) Δ^π in each industry sector:

$$(19) \qquad \Delta_j^\pi = p_j - A_j \prod_{i=1}^{N} \left(p_i / \beta_{ij} \right)^{\beta_{ij}} \prod_{f=1}^{F} \left(w_f / \gamma_{fj} \right)^{\gamma_{fj}} \, .$$

Finally, the income balance condition (16) can be re-written in terms of the excess of income over returns to the agent's endowment of primary factors, Δ^m:

$$(20) \qquad \Delta^m = \sum_{f=1}^{F} w_f V_f - m \, .$$

General equilibrium is thus the joint minimization of Δ^C, Δ^F, Δ^π and Δ^m.

5. The Formulation, Calibration and Solution of a CGE Model

5.1. Model Formulation

The way in which a CGE model is solved for an equilibrium can now be readily understood. To obtain a solution, the model uses eqs. (17)-(20). These expressions are formulated a system of $2N + F$ equations in $2N + F$ unknowns: an N-vector of industry output- or "activity" levels $\mathbf{y} = [y_1, \cdots, y_N]$, an N-vector of commodity prices $\mathbf{p} = [p_1, \cdots, p_N]$, an F-vector of primary factor prices $\mathbf{w} = [w_1, \cdots, w_F]$ and a scalar income level m. The problem of finding the vector of activity levels and prices that supports general equilibrium therefore consists of choosing values for these variables to solve the problem

$$(21) \qquad \xi(\mathbf{z}) = \mathbf{0} \, ,$$

in which $\mathbf{z} = [\mathbf{p}, \mathbf{w}, \mathbf{y}, m]'$ is the vector of stacked prices, activity levels and level of income, and $\xi(\cdot) = [\boldsymbol{\Delta}^C, \boldsymbol{\Delta}^F, \boldsymbol{\Delta}^\pi, \Delta^m]'$ is the system of stacked pseudo-excess demand equations, which forms the production-inclusive pseudo-excess demand correspondence of the economy.

Eq. (21) is the expression of general equilibrium in a complementarity format, so named because of the important complementarity that exists between prices and excess demands, and between activity levels and profits, and that is a critical feature of general equilibrium. For the equilibrium above to be economically meaningful, prices, activity levels and income are all positive and finite ($\mathbf{0} \leq \mathbf{z} < \infty$). In the limit, as $\mathbf{z}$ approaches zero, eqs. (17), (19) and (20) all approach zero, and eq. (18) tends to $-V_f$, implying that $\xi(\mathbf{0}) = [\mathbf{0}, -\mathbf{V}, \mathbf{0}, 0]' \leq \mathbf{0}$. If $\mathbf{z^*}$ is a vector of prices and activity and income levels that supports general equilibrium, it must be the case that $\mathbf{0} \le \mathbf{z^*}$ and $\xi(\mathbf{z^*}) = \mathbf{0}$. Thus, the problem in eq. (21) may be compactly re-specified as one of finding

(22) $\quad \mathbf{z} \ge \mathbf{0}$ subject to $\quad \xi(\mathbf{z}) \ge \mathbf{0}, \quad \mathbf{z}'\xi(\mathbf{z}) = \mathbf{0}$,

which is a mathematical statement of Walras' Law that the sum of the values of market demands equal to the sum of the values of market supplies.[7]

5.2. Numerical Calibration Using the SAM

Even with a specification of preferences and technology that is as simple as the C-D, the problem in eq. (22) is still highly non-linear, with the result that there is no closed-form analytical solution for **z**. This is the reason for the "C" in CGE models: general equilibrium systems with realistic utility and production functions must be calibrated on a SAM of the kind discussed in section 3, generating a numerical optimization problem that can be solved using optimization techniques.

Numerical calibration is easily accomplished in the C-D economy. The crucial step in this regard is to compare eqs. (1)-(4) with eqs. (13)-(16). The pairs (1) and (13), (2) and (14), (3) and (15), and (4) and (16) exhibit a striking symmetry. In particular, the elements of each pair are equivalent if $\pi_j = 0$ (zero profit, which we assume), $p_i x_{ij} = \bar{x}_{ij}$ and $w_f v_{fj} = \bar{v}_{fj}$. Therefore, a fundamental equivalence may be drawn between the equations in a CGE model and the benchmark flows of value in a SAM by assuming that in the benchmark year all prices are equal to unity.

The foregoing observation is the essence of the simplest calibration procedure by which a CGE model is "fit" to the benchmark equilibrium recorded in a SAM.[8] All prices are treated as index numbers with a value of unity in the benchmark, and all value flows in the SAM are treated as benchmark quantities. These assumptions allow the technical coefficients and elasticity parameters of the utility and production functions to be solved for directly (Mansur and Whalley 1983):

(23) $\quad \alpha_i = \bar{g}_{iC} / \bar{G}_C$,

(24) $\quad A_C = \bar{G}_C \Big/ \left(\prod_{i=1}^{N} \bar{g}_{iC}^{\alpha_i} \right)$,

(25) $\quad \beta_{ij} = \bar{x}_{ij} / \bar{y}_j$,

(26) $\quad \gamma_{fj} = \bar{v}_{fj} / \bar{y}_j$,

(27) $\quad A_j = \bar{y}_j \Big/ \left(\prod_{i=1}^{N} \bar{x}_{ij}^{\beta_{ij}} \prod_{f=1}^{F} \bar{v}_{fj}^{\gamma_{fj}} \right)$,

(28) $\quad s_i = \bar{g}_{iS}$,

(29) $\quad V_f = \bar{V}_f$

[7] See, e.g. Varian (1992: 343).

[8] For an alternative procedure, see Kehoe (1998a). In the general case a CGE model's production and consumption technologies are neither Leontief nor Cobb-Douglas. Then, in order to calibrate the model the value of the elasticities of substitution must be assumed by the modeler, as there are more estimated parameters than model equations, making the calibration problem under-determined. The econometric approach to calibration (e.g., Jorgenson 1984) circumvents much of the potential ad-hocracy in this process, but is data intensive, requiring time-series of social accounting matrices which are often not available. See Dawkins et al (2001) for an excellent survey of these issues.

and

(30) $\overline{m} = \sum_{f=1}^{F} \overline{V}_f$.

Having specified these values for the model's coefficients, solving the numerical problem in eq. (22) will then set the quantities of the variables in the C-D economy equal to the values of the corresponding flows in the SAM (i.e., $x_{ij} = \overline{x}_{ij}$, $v_{fj} = \overline{v}_{fj}$ and $c_i = \overline{g}_{iC}$), replicating the benchmark equilibrium.[9]

5.3. The Solution of a CGE Model in a Complementarity Format

The calibration procedure transforms (22) into a square system of numerical equations known as a *nonlinear complementarity problem* or NCP (Ferris and Pang, 1997), which may be solved using algorithms that are now routinely embodied in modern, commercially-available software systems for optimization.[10] Mathiesen (1985a,b) and Rutherford (1987) describe the basic approach, which is similar to a Newton-type steepest-descent optimization algorithm (Kehoe 1991: 2068-2072). The algorithm iteratively solves a sequence of linear complementarity problems or LCPs (Cottle et al 1992), each of which is a first-order Taylor series expansion of the non-linear function ξ. The LCP solved at each iteration is thus one of finding

(31) $\mathbf{z} \geq \mathbf{0}$ subject to $\mathbf{q} + \mathbf{M}\mathbf{z} \geq \mathbf{0}, \quad \mathbf{z}'(\mathbf{q} + \mathbf{M}\mathbf{z}) = \mathbf{0}$,

where, linearizing ξ around $\mathbf{z}_{(k)}$, the state vector of prices activity levels and income at iteration k, $\mathbf{q}(\mathbf{z}_{(k)}) = \nabla\xi(\mathbf{z}_{(k)})\mathbf{z}_{(k)} - \xi(\mathbf{z}_{(k)})$ and $\mathbf{M}(\mathbf{z}_{(k)}) = \nabla\xi(\mathbf{z}_{(k)})$. Then, starting from an initial point $\mathbf{z}_{(0)}$, the solution of the problem (23) at the k^{th} iteration $\mathbf{z}^*_{(k)}$ updates the value of $\mathbf{z}$ according to:

(32) $\mathbf{z}_{(k+1)} = \mu_{(k)}\mathbf{z}^*_{(k)} + (1 - \mu_{(k)})\mathbf{z}_{(k)}$,

where the parameter $\mu_{(k)}$ controls the length of the forward step in $\mathbf{z}$ that the model takes at each iteration. The convergence criterion for the algorithm consisting of eqs. (31) and (32) is just the numerical analogue of eq. (21): $\left\|\xi(\mathbf{z}_{(k)})\right\| < \varpi$, in which the scalar parameter ϖ is the maximum tolerance level of excess demands, profits, or income at which the algorithm is deemed by the analyst to have converged to an equilibrium.[11]

5.4. Existence and Uniqueness of Equilibrium

The foregoing exposition raises the question of how good are CGE models at finding an equilibrium. Experience with the routine use of CGE models calibrated on real-world economic data to solve for equilibria with a variety of price and quantity distortions would seem to indicate that the procedures outlined above are robust. However, an answer to this question is both involved and elusive, as it hinges on three important underlying issues which span the theoretical

---

[9] This calibration technique is equivalent to expressing the utility and production functions in *calibrated share form* (see Boehringer et al 2003: Tables 1 and 2).

[10] Foremost among these is the PATH solver (Dirkse and Ferris 1995). It is especially powerful when used in combination with other software tools such as the Generalized Algebraic Modeling System (GAMS) numerical language (Brooke et al. 1999) and the MPSGE pre-processing subsystem (Rutherford, 1995, 1999), which automatically calibrates the technical coefficients in equations (9), (11) and (12) based on a SAM provided by the user, and formulates the general equilibrium problem as square system of nonlinear equations which is solved as an NCP.

[11] In the operations research literature there are by now numerous refinements to this approach, generally based on the path-following methods described in Kehoe (1991: 2061-2065). See Dirkse and Ferris (1995), Ferris et al (2002) and Ferris and Kanzow (2002: §4) for details of the algorithms and discussions of their convergence properties.

and empirical literatures on general equilibrium: the existence, uniqueness, and stability of equilibrium. Clearly, these are desirable attributes of a CGE model, as they imply that its solutions are predictable, replicable and robust to perturbations along the path to convergence (e.g., through changes in $\mu_{(k)}$).

Textbook treatments of the theory of general equilibrium emphasize two properties that ξ should satisfy. The first is the weak axiom of revealed preference (WARP), whereby an economy with multiple households exhibits a stable preference ordering over consumption bundles in the space of all possible prices and income levels, ruling out the potential for non-homothetic shifts in households' consumption vectors if incomes change but prices stay the same. A sufficient condition for this property to hold is that households' preferences admit aggregation up to well-behaved community utility function, which is the representative agent assumption. The second property is gross substitutability (GS), where the aggregate demand for any commodity or factor is non-decreasing in the prices of all other goods and factors. Where this holds, a vector of equilibrium prices exists and is unique up to scalar multiple (Varian 1992).

One can think of the foregoing conditions as economic interpretations of the sufficient conditions for a unique solution to (21). From a mathematical standpoint, a (locally) unique solution for **z** can be recovered from the inverse of the pseudo-excess demand correspondence. The inverse function theorem implies that a sufficient condition for ξ to be invertible is that its jacobian is non-singular, which require $-\nabla\xi$ to be positive semi-definite. Loosely speaking, GS and WARP both imply that the determinant of $-\nabla\xi$ is non-negative—generally that it is positive (Kehoe 1985).[12] But in real-world applied policy models there are often many sectors and agents that are each specified using algebraically complex nested utility or production functions, making ξ and its jacobian sufficiently large and non-linear to render closed-form analytical proofs of this condition impossible. An emerging area of computational economic research is the development of algorithms to test the positive determinant property at each iteration step of the numerical sub-problem.

Theoretical studies of general equilibrium have focused on finding the least restrictive conditions on ξ that enable WARP and/or GS to ensure uniqueness, and have largely circumvented the details of algebraic functional forms employed in applied models. The signal exception is Mas-Colell (1991), who proves that so-called "super Cobb-Douglas economies"—i.e., those with constant elasticity of substitution (CES) utility and production functions whose elasticities of substitution are greater than or equal to one—are guaranteed to have a unique equilibrium in the absence of taxes and other distortions.[13] In the context of the present analysis, this result is both directly relevant and encouraging. However, it is tempered by evidence that distortions can have the effect of inducing multiple equilibria, even in models with a representative consumer and convex production technologies (Foster and Sonnenschein 1970; Hatta 1977). Although this finding seems to turn on the fact that at least one commodity is an inferior good (Kehoe 1985)—a rarity in applied work—the potential for distortions to introduce instability is worrying because, as the next section will elaborate, CGE models are the workhorse of the empirical analysis of the incidence and distortionary effects of policies.

Tests of the theory have focused on the construction, diagnosis and analysis of multiple equilibria in simple, highly stylized CGE models. Kehoe (1998b) analyzes a model that has two

---

[12] This condition is satisfied when the diagonal elements of this matrix are non-negative and the off-diagonal elements are non-positive. That this is implied by WARP and GS is strictly only correct for an excess demand correspondence defined solely on prices. The addition of activity and income levels in **z** introduces complications.
[13] See pp. 291-294, especially Theorem 3.

consumers, each with Cobb-Douglas preferences, and four commodities produced with an activity analysis technology. The model's excess demand correspondence satisfies the GS property, yet it exhibits three equilibria, indicating the minor role played by the GS condition in determining the equilibrium of economies with production. However, changing the model's production functions to Cobb-Douglas technologies collapses the number of equilibria to one, confirming Mas-Colell's (1991) result. Kehoe (1998b) concludes that the only guarantees of uniqueness are the very restrictive conditions of a representative consumer and complete reversibility of production. The latter condition implies that the supply side of the economy is an input-output system in which there is no joint production, and where consumers possess no initial holdings of produced goods but do hold initial endowments of at least one non-reproducible commodity or factor.

However, it is still questionable whether these conditions still ensure uniqueness in the presence of tax distortions, because of the complex influence of the flows of revenue that taxes generate on the representative agent's income and its feedback on the vector of commodity demands and producers' activity levels. Whalley and Zhang (2002) present examples of pure exchange economies that have either a unique equilibrium without taxes and multiple equilibria with taxes, or multiple equilibria without taxes and a unique equilibrium with the introduction of a small tax. Kehoe (1998b) shows that sufficient condition for uniqueness in the presence of a tax distortion is that the weighted sum of the income effects, in which the weights are given by the "efficiency" (i.e., net-of-tax producer) prices, must be positive. In the presence of pre-existing distortions in the benchmark SAM, the fact that calibration of the model will set all prices to unity makes this condition easy to verify. However, if taxes are specified as algebraic functions of variables within the model, this condition may be virtually impossible to check prior to actually running the model and inspecting the equilibrium to which it converges. The intuition is that, with a specified revenue requirement and endogenous taxes, even models that satisfy all of the other prerequisites for uniqueness will have a Laffer curve that yields two equilibria, one in which the tax rate is high and the other in which it is low.[14] All this suggests the lurking possibility that multiplicity may be induced by changes in tax parameters, and may be difficult to predict ex ante, or even detect.

It is therefore unsurprising that tests of multiplicity of equilibria in real-world CGE models are few and far between. Kehoe and Whalley (1985) find no evidence of multiplicity in the Fullerton et al (1981) and Kehoe-Serra-Puche (1983) tax models, while reports of multiple equilibria are restricted to models with increasing returns (Mercenier 1995; Denny et al 1997). Research in this area is ongoing, focusing on translating theoretical results into numerical diagnostic tools (e.g., Dakhlia 1999). But without the ability to test for—or remedy—the problem of multiple equilibria, most applied modelers proceed on the assumption that the solutions generated by their simulations are unique and stable. As Dakhlia (1999) points out, whether this is in fact the case, or whether multiplicity usually just goes undetected, is still open. Thus, to return to the question with which this section began, the C-D economy model has nice properties that guarantee a unique equilibrium once there are no taxes or subsidies (Mas-Colell 1991). Furthermore, if there are exogenous distortions the equilibrium will still be unique (Kehoe 1985), but this result is not assured in the presence of distortions that are endogenous. The remainder of the paper deals with the effects of exogenous distortions in more detail.

[14] I thank Tim Kehoe for providing me with this insight.

6. Policy Analysis: The General Equilibrium Effects of Tax Distortions

CGE models are the primary tool for analyzing the impacts across multiple markets of changes in one or more policy variables. These are model parameters that are either price-based (e.g., taxes and subsidies) or quantity-based (e.g., constraints on demand and/or supply), and whose values are often exogenously specified by the analyst. When the economy is initially at its unfettered equilibrium, the perturbation in prices, activity levels and demands caused by a change in the values of these parameters induces convergence to a new, distorted equilibrium. By comparing the pre- and post-change equilibrium vectors of prices, activity levels, demands and income levels, the policy may be evaluated, subject to the caveats of the accuracy and realism of the model's assumptions.

The advantage of this approach is its ability to measure policies' ultimate impact on aggregate welfare in a theoretically consistent way, by quantifying the change in the income and consumption of the representative agent that result from the interactions and feedbacks among all of the markets in the economy. Yet this very facility is at the root of the "black box" criticism raised in the introduction, as it creates the temptation for some policymakers and analysts to treat CGE models as a sort of economic crystal ball. Yet CGE models' usefulness in policy analysis owes less to their predictive accuracy, and more to their ability to shed light on the economic mechanisms through which price and quantity adjustments are transmitted among markets. Therefore, while on a superficial level CGE models can be thought of as a pseudo-empirical tool to quantify the impacts of imposing or removing policy distortions in a "what-if" manner, they should properly be regarded as computational laboratories within which to analyze the dynamics of the economic interactions from which these impacts arise (Francois 2001). The black box critique is therefore concerned with the fact that the analysis does not account for the linkages between simulation results and the characteristics and assumptions of the models that generate them, and less with the models themselves.

We illustrate these issues by applying the C-D economy of the preceding section to the analysis of the incidence and distortionary effects of taxation. Within CGE models, taxes are typically specified in an ad-valorem fashion, whereby a tax at a given rate determines the fractional increase in the price level of the taxed commodity. For example, an ad-valorem tax at rate τ on the output of industry j drives a wedge between the producer price of output p_j and the consumer price $(1 + \tau)\, p_j$, in the process generating revenue from y_j units of output in the amount of $\tau\, p_j\, y_j$. A subsidy that lowers the price may be also incorporated in this way, by specifying $\tau < 0$.

Conceptually, there are four types of markets in the economy in which ad-valorem taxes or subsidies can be levied: the market for the output of each industry sector, the market for consumption goods, and the markets for inputs to production of intermediate goods and primary factors in each industry. Let the tax or subsidy rates that correspond to each of these markets be denoted by τ_j^Y, τ_i^C, τ_{ij}^X and τ_{fj}^V, respectively.[15] Then, in light of these distortions the representative agent's problem becomes

$$(8') \qquad \max_{c_i}\, p_U U - \sum_{i=1}^{N} (1+\tau_i^C)(1+\tau_i^Y)\, p_i c_i$$

[15] The superscripts on the tax rates are meant to reflect the nomenclature used thus far to identify different economic quantities: output Y, consumption C, intermediate inputs X, and factor inputs V. Note that the tax on consumption can be generalized to a matrix of taxes on several different final demand activities, principally investment and, in open-economy models, imports and exports. Since the setup of the C-D economy model in section 4 treats saving and investment as exogenously fixed, these additional distortions will not be discussed further.

subject to the constraint of the C-D utility function, and each producer's problem is

(10') $$\max_{x_{ij}, v_{fj}} \pi_j = p_j y_j - \sum_{i=1}^{N} (1+\tau_{ij}^X)(1+\tau_i^Y) p_i x_{ij} - \sum_{f=1}^{F} (1+\tau_{fj}^V) w_f v_{fj}$$

subject to the constraint of the C-D production function, which modify the commodity and factor demand functions in eqs. (9), (11) and (12) as follows:

(9') $$c_i = \alpha_i \frac{\left(m - \sum_{i=1}^{N} (1+\tau_i^Y) p_i s_i \right)}{(1+\tau_i^C)(1+\tau_i^Y) p_i},$$

(11') $$x_{ij} = \beta_{ij} \frac{p_j y_j}{(1+\tau_{ij}^X)(1+\tau_i^Y) p_i},$$

and

(12') $$v_{fj} = \gamma_{fj} \frac{p_j y_j}{(1+\tau_{fj}^V) w_f}.$$

Each of the taxes (subsidies) outlined above generates a positive (negative) revenue stream that from an accounting perspective must both increment (decrement) the income of some agent and negatively (positively) affect the absorption and generation of a commodity or factor. Representative-agent models often simulate this phenomenon by treating the government as a passive entity that collects tax revenue and immediately recycles it to the single household as a lump-sum supplement to the income from factor returns. The model may therefore omit government as an explicit sector altogether, simply specifying taxes as transfers of purchasing power from producers to the representative agent. The latter's income then becomes:

(16') $$m = \sum_{f=1}^{F} w_f V_f + \underbrace{\sum_{j=1}^{N} \tau_j^Y p_j y_j}_{\substack{\text{Output} \\ \text{tax revenue}}} + \underbrace{\sum_{i=1}^{N} \tau_i^C p_i c_i}_{\substack{\text{Consumption} \\ \text{tax revenue}}} + \underbrace{\sum_{i=1}^{N} \sum_{j=1}^{N} \tau_{ij}^X p_i x_{ij}}_{\substack{\text{Intermediate input} \\ \text{tax revenue}}} + \underbrace{\sum_{f=1}^{F} \sum_{j=1}^{N} \tau_{fj}^V w_f v_{fj}}_{\substack{\text{Factor} \\ \text{tax revenue}}}.$$

This expression, along with eqs. (9') and (11'), when substituted into (13), eq. (12'), when substituted into (14), and eqs. (11') and (12'), when substituted into the production function, form the basis for a new excess demand correspondence that, when cast in the format of eq. (22) may be solved to yield a new, tariff-ridden equilibrium. The vector of prices and the allocation of commodity purchases and factor inputs thus solved for may then be compared with those of the original, benchmark equilibrium. Note that, for positive tax rates the tax revenue terms in (16') will be all positive, satisfying the condition for uniqueness discussed in section 5.4.

Consumption and income are the most useful variables in ascertaining the welfare effects of distortions. In particular, the change in the value of the aggregate consumption of the representative agent as a result of the tax or subsidy is nothing more than equivalent variation: the loss of value of consumption occasioned by the effects of the distortion on relative prices. The welfare effect of a single tax or subsidy thus depends on the interaction of a number of factors: the level of the tax and the distribution of other taxes and subsidies across all markets in the economy, the characteristics of the particular market in which the tax is levied, the linkages between this market and the others in the economy, and the values of the vectors of calibrated parameters $\mathbf{A}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

Herein lies the kernel of truth to the black box criticism discussed earlier. Because of the non-linearity of the general equilibrium problem in eq. (22), it is often difficult to intuit what all the impacts of a single distortion will be, even in models with only a modest number of sectors and/or households. Further, to sort through and understand the web of interactions that give rise to the post-tax equilibrium may require the analyst to undertake a significant amount ex post analysis and testing. This point is illustrated in the following section.

7. Taxes in a 2 × 2 × 1 Cobb-Douglas Economy

The methods by which a CGE model is formulated, solved, and then used to analyze the effects of taxes, are best conveyed by means of a simple, practical example. This section explores the impacts of distortions in a C-D economy of the kind dealt with thus far, for the purpose of elucidating the propagation among markets of taxes' general equilibrium effects on prices and quantities.

The model is a C-D economy in which there is a single representative agent, two industries ($j = \{1, 2\}$), each of which produces a single output good ($i = \{1, 2\}$), and two primary factors of production, labor L and capital K ($f = \{L, K\}$). A SAM for this economy is shown in Figure 3. These data represent the benchmark equilibrium for a CGE model whose excess demand correspondence is eqs. (17)-(20), the parameters of which are calibrated according to eqs. (23)-(29).[16] There are no taxes in the benchmark equilibrium recorded in Figure 3, so that the values of τ_j^Y, τ_i^C, τ_{ij}^X and τ_{fj}^V are initially zero.

This economy is sufficiently simple that specifying positive values any of the distortion parameters τ will generate general equilibrium effects across all markets that can be both fully characterized and intuitively explained. This is illustrated by imposing a 50 percent tax in each of the different markets in the economy, resulting in a series of distorted equilibria that differ with respect to the economy's benchmark state in terms of commodity and factor prices, quantities of commodity demand and output, the inter-industry distribution of primary factor uses, and the value of consumption by the representative agent. The results are shown in Table 1. Throughout, the measure of welfare is the aggregate expenditure of the representative agent on consumption, or income net of saving ($m - s$). Equivalent variation (EV) is measured as the percentage change in this quantity from its benchmark level.[17]

7.1. Aggregate Commodity Taxes

Taxes on the output of either industry create the largest market distortions and have the largest negative effect on welfare, for the simple reason that the resulting price effects ripple throughout the entire economy. The tax increases the relative price of the commodity on which it is levied, which results in a reduction in the demand for its use by the representative agent for consumption and by the non-taxed industry for intermediate input. For the commodity market to clear, the activity of the taxed sector must decline relative to its benchmark output level, which in turn reduces the taxed industry's demand for intermediate inputs (both own-supplied and produced by the non-taxed sector), and primary factors. For the representative agent's factor endowment to be exhausted, absorption of labor and capital inputs by the non-taxed industry must increase to the point where it just picks up the slack between factor demand and supply. This in turn causes an increase in this sector's activity relative to its benchmark output level, and

[16] Appendix C gives the computer code for the model written in GAMS/MPSGE syntax.

[17] Recall that the value of p_U is fixed at unity as the numeraire price. This calculation therefore measures the effect of the tax in terms of the change in the quantity of aggregate consumption, measured at pre-tax prices.

a concomitant rise in the consumption (and fall in the price) of the non-taxed good to clear the goods market. Additionally, the reduction in the wage and the capital rental rate necessary for factor market clearance (a consequence of their inelastic supply) precipitates a decline in the income of the representative agent that is only partially offset by the revenue from the tax. The result is a decline in the aggregate consumption expenditure of the representative agent, and a decrease in welfare.

A few further points deserve mention. The relative intensities with which activities use different inputs are crucial determinants of the pattern of general equilibrium effects. If the production of a good is relatively intensive in the use of a particular input (e.g., sector 1's use of labor or sector 2's use of capital), then a tax on the output of that good will require a relatively larger reduction in price of that input to clear the market. It is also interesting to note that once these general equilibrium interactions are fully accounted for, the price of the taxed commodity increases relative to its benchmark level, but not by the full amount of the tax. This highlights the importance of general equilibrium interactions, particularly the compensating income effects of recycling tax revenue to the representative agent. Also, the welfare loss precipitated by a tax on commodity 2 is larger, despite the fact that it is the smaller of the two industries, because its share of consumption is larger.[18]

7.2. Taxes on Consumption

A consumption tax increases the relative price of the taxed good to the representative agent, causing her consumption of that good to decrease. The price and output levels of the taxed good must then fall for the market to clear, leading to a reduction in the demand for intermediate goods and primary factors in that industry, and, by the mechanisms described above, an expansion in the output of the non-taxed industry, a reduction in primary factor prices, and a decline in the income of the representative agent. Compared to a tax on the output of a good, taxing only the portion of output that is consumed causes a much smaller reduction in both the level of production activity and the less severe knock-on general equilibrium effects, thus precipitating a much smaller welfare loss.

7.3. Aggregate Factor Taxes

Taxes on the simultaneous use of primary factors in both industries have the smallest distortionary impacts. The reason is that in the simulated economy labor and capital are both inelastically supplied by the representative agent. Thus, instead of precipitating changes in industries' aggregate demands for these inputs, the tax must be accommodated by a downward adjustment in the net-of-tax price of the taxed factor that enables the market to clear. Further, because the revenue from factor taxes is recycled to the representative agent in a lump-sum fashion, this additional income exactly balances the loss of income from the reduction in the net-of-tax factor price. Industries see the same prices, the representative agent sees the same level of income, and the resulting equilibrium is indistinguishable from the business-as-usual baseline.[19]

7.4. Sector-Specific Taxes on Intermediate Inputs

Taxes on intermediate inputs tend to have effects that are localized within the producing sector in which the tax is levied, and therefore exert only small impacts on aggregate output,

[18] All of these statements can be easily verified by altering the pattern of flows in the SAM to simulate different input intensities, and re-running the model.
[19] Note that these results would differ markedly if labor and capital were in elastic supply.

income, and welfare. The effects on the prices of primary factor inputs and output are negligible in both sectors. In each industry, the tax precipitates a decline in significant output only if it is levied on that industry's own use of its output as an intermediate input, on the use of that industry's output as an intermediate input to another sector, or on the industry's use of another sector's output. Imposing a tax on an industry's use of its own output has negligible spillover effects on the output of the non-taxed sector. But in the industry where the tax is levied, output falls, driving down its demand for factor inputs, whose prices must decline to clear the market, and whose excess supply is absorbed by the non-taxed sector. And although consumption of the output of the non-taxed industry rises—as a result of consumer substitution in response to the fall in its price relative to the unit cost of production in the taxed sector, the income effects of revenue recycling are insufficient to restore overall demand for the output of the non-taxed industry to its benchmark level.

7.5. Sector-Specific Factor Taxes

The distortionary effects of taxes on the factors employed by each sector are similarly localized. The gross-of-tax price factor increase as a result of the tax reduces the demand for that factor in the industry where the tax is levied, precipitating a decline in the factor's the net-of-tax price. As a result of the substitution effect, that industry's use of the non-taxed factor increases. Unit costs in the taxed sector also increase, causing the price of that sector's output to rise and the quantity of its output to fall. Substitution at the level of the consumer causes a reduction in demand for the output of the industry in which the tax is levied, and a concomitant increase in demand for the output of the non-taxed sector. Overall, taxing labor gives rise to larger welfare losses, as it is a larger overall share of value added.

8. A More Realistic Application: The Impacts of Carbon Taxes on the U.S. Economy

This final section presents a more realistic application of methods for formulating, calibrating and solving a CGE model, this time using actual economic data to analyze a real-world policy problem. Its focus is the economic impacts of policy to mitigate the emission of heat-trapping greenhouse gases (GHGs) that contribute to global warming, an issue that is both one of the foremost policy problems of our time and fertile ground for the application of CGE modeling techniques. The most important GHG is carbon dioxide (CO_2), whose anthropogenic emission is largely due to the combustion of carbon-rich fossil fuels. On the supply side of the economy, fossil fuels are the sole large-scale source of energy, while on the demand side, energy is employed as an input to virtually every activity, raising concerns that even modest taxes or quantitative limits on CO_2 emissions will precipitate large increases in energy prices, reductions in energy use, and declines in economic output and welfare. The economy-wide character of the issue implies that elucidating the impacts of carbon taxes requires the kind of analysis for which CGE models are particularly well suited.[20] This section therefore adapts the model of the C-D economy to this task.

8.1. Model Setup and Calibration

Structurally, the model to be employed is identical to that in the previous section; here, however, its dimensions are larger. The demand side of the economy is modeled as a representative agent that demands commodities to satisfy three categories of final uses: consumption, investment, and net exports, the latter two of which are held fixed for ease of

[20] See e.g., the analyses that employ CGE models in Weyant (1999).

exposition and analysis. The supply side of the economy is modeled as seven aggregate sectors: coal mining, crude oil and gas, natural gas distribution, refined petroleum, electric power, energy-intensive manufacturing (an amalgam of the chemical, ferrous and non-ferrous metal, pulp and paper, and stone, clay and glass industries), transportation, and a composite of the remaining manufacturing, service, and primary extractive industries in the economy. Labor and capital are the primary factors, as before. In line with the present application, this disaggregation scheme models the energy sectors of the economy in detail, while aggregating a large number of other activities that, although being far more important contributors to gross output, are not germane to the climate problem.

The SAM used to calibrate this model is constructed from the BEA's 94-sector Make of Commodities by Industries and Use of Commodities by Industries tables for the year 1999, using the industry technology assumption (for details see, e.g., Reinert and Roland-Holst 1992), and its components of value added are disaggregated using data on industries' shares of labor, capital, taxes and subsidies in GDP from the BEA's GDP by Industry accounts. The resulting benchmark flow table is aggregated up to seven sectoral groupings outlined above, scaled to approximate the U.S. economy in the year 2000 using the growth rate of real GDP from 1999-2000, and deflated to year 2000 using the GDP deflator from the NIPAs.

Figure 4 shows the final SAM, whose structure is similar to Figure 2(b) in terms of the presence of an additional *N*-vector $\overline{\mathbf{T}}^Y$ of benchmark payments of net taxes on industry outputs. These distortions affect the benchmark equilibrium, and therefore need to be taken into account in the calibrating the model. To do so, the first step is to work out the tax and subsidy rates that are implied by the benchmark flows of tax payments in the SAM. The payments for taxes on the outputs of industry sectors $\bar{t}_j^Y$ denotes the component of the value of the output of each industry $\bar{y}_j$ paid to the government as tax revenue. Specifying these distortions in ad-valorem terms, the average benchmark tax rate in sector *j* is $\bar{\tau}_j^Y = \bar{t}_j^Y / \bar{y}_j$, and the fact that the SAM only contains benchmark taxes on output implies that $\bar{\tau}_i^C = \bar{\tau}_{ij}^X = \bar{\tau}_{fj}^V = 0$.

The second step is to utilize the distortion-inclusive commodity and factor demand equations developed in section 6 to compute the technical coefficients and elasticity parameters of the utility and production functions. Then, setting all prices to unity and using the flows in the SAM as benchmark quantities in eqs. (9'), (11'), (12') and (16') yields eqs. (23)-(24), (26)-(29) and the modified calibration equations:

(25') $\beta_{ij} = (1+\bar{\tau}_i^Y)\bar{x}_{ij} / \bar{y}_j$,

and

(30') $\overline{m} = \sum_{f=1}^{F} \overline{V}_f + \sum_{j=1}^{N} \bar{t}_j^Y$.

Solving eq. (22) with these parameter values replicates the distorted equilibrium in Figure 4.

For simplicity, taxes are modeled as lump-sum transfers per the discussion in section 6. The model simulates the effect of imposing a range of additional taxes on emissions of CO_2, which is a by-product of production and consumption activities. To calculate the burden of these new taxes on industries and the representative agent, it is necessary to establish the relationship between the levels of production and demand activities and the quantity of emissions. The simplest way of doing this is to assume a fixed stoichiometric relationship between the aggregate demand for fossil fuel commodities *e* ($e \subset i$) in which carbon is embodied (i.e., coal, refined petroleum and natural gas) and the quantity of atmospheric CO_2 emissions that result from their use.

The result is a set of commodity-specific emission coefficients ε_e, which when multiplied by each fossil fuel's aggregate demand in the SAM, reproduces the economy's emissions of CO_2 in the benchmark year.[21] A tax on carbon τ^{Carb} therefore results in a set of commodity taxes that are differentiated by energy goods' carbon contents, and acts to increase the gross-of-ad-valorem-tax price of each fossil fuel $(1+\tau_e^Y)p_e$ by a further margin $\tau^{Carb}\varepsilon_e$. The model is simulated to reproduce the benchmark as a baseline no-policy case, with the imposition of carbon taxes at levels of \$50, \$100, \$150 and \$200 per ton of carbon.[22,23]

8.2. Results and Discussion

The previous section illustrated CGE models' utility in elucidating the impacts of distortions on prices and quantities across all of the markets in the economy. This is also true of the present example, for which the price and quantity impacts of carbon taxes are detailed in Table 2. The top panel shows that a \$50/ton carbon tax raises the consumer prices of petroleum and natural gas by 20 percent and makes coal almost one and a half times more expensive, while a \$200/ton increases the prices of coal and oil by three-quarters and the price of coal by a factor of more than five and a half.

These prices changes induce large adjustments in the quantities of fossil fuels used as inputs by producers and households, where inter-fuel substitution enables reductions in demand to be concentrated in the most carbon-intensive energy source, coal. Thus, in the second to the fifth panels of Table 2, all sectors see declines in coal use by 60-97 percent, while in the non-fossil-fuel sectors, demands for both petroleum and natural gas decline by 17-46 percent, and electricity demand shrinks by only 6-15 percent. In these latter sectors of the economy, substitution of non-energy inputs for fossil fuels mitigates the transmission of the reductions in output of primary energy sectors. The sixth panel in the table shows that these are on the order of 22-52 percent for petroleum and natural gas, and 59-83 percent for coal, and 19-50 percent for crude petroleum mining. As a result, the level of output falls by 7-14 percent in electric power, 1-4 percent in energy-intensive industries and transportation, and only 0.1-0.4 percent in the rest of the economy. The final panel shows that these changes in activity levels are correspond closely to changes in the consumption of the corresponding commodities by the representative agent.

CGE analyses also facilitate insights into the impacts of environmental policy interventions on pollution. In this example, CO_2 emissions and their abatement may be computed by applying the benchmark emission coefficients ε_e to the new levels of aggregate demand for fossil fuels in the distorted equilibria. The emissions from each sector are shown in Figure 5,

---

[21] For coal, petroleum and natural gas, emissions of carbon in the base year were divided by commodity use in the SAM (calculated as gross output – net exports). CO_2 emissions in the year 2000 from coal, petroleum and natural gas are 2,112, 2,439 and 1,244 MT, respectively (DOE 2003), while the aggregate use of these commodities in the SAM is 21.8, 186.5 and 107.1 billion dollars, respectively. The emission coefficients for coal, petroleum and natural gas are thus 0.097, 0.012 and 0.013 tons of CO_2 per dollar, respectively.

[22] A potential source of confusion in that GHG taxes are usually specified in units of carbon while environmental statistics usually account for GHG emissions in units of CO_2. The ratio of these substances' molecular weights (0.273 tons of carbon per ton of CO_2) establishes an equivalency between the two measures. Thus, the values of τ^{Carb} above are equivalent to taxes on CO_2 that are less than one-third as large: \$13.6, \$27.3, \$40.9 and \$54.5 per ton of CO_2.

[23] The model code in GAMS/MPSGE syntax is shown in Appendix D. The results that it generates differ slightly from those in the paper as the latter employ a SAM with higher numerical precision (six significant digits).

which shows that CO_2 emissions could be halved from the BaU level of 5796 MT by a carbon tax of $100/ton, and that a $200/ton tax could cut emissions by almost two-thirds. Figure 6 plots the sectoral marginal abatement cost (MAC) curves derived from the model's solution. The MAC curves are well-behaved (i.e., continuous, smooth, and convex to the business-as-usual origin), which is a reflection of the homotheticity of the model's utility and production.

Figure 6 shows that the bulk of abatement occurs in the rest-of-economy, household and electric power sectors, with the first two sectors together being responsible for as large a reduction of emissions as the latter (approximately 800-1300 MT). Less than half as much abatement (350-500 MT) takes place in the coal mining and energy intensive industries, with a further 66-106 percent of that (235-530 MT) being generated by the natural gas, refining and transportation sectors, and only a small quantity of emission reductions (25-53 MT) coming directly from the mining of fossil fuels. These results indicate that while there may be substantial low-cost abatement opportunities (less than $50/ton) in many industries, incremental emission reductions are likely to be exhausted at tax levels of greater than $100/ton in all but the final consumption, rest-of-economy and electric power sectors.

The utility of CGE analyses in analyzing incidence of taxes is illustrated in Table 3. For each sector, the direct cost of abatement is approximated by the area under the MAC curve in Figure 6 that corresponds to the level of the tax. These costs are on the order of 6-10 percent of the value of benchmark output in the coal industry, 2-7 percent in electric power, 0.5-3.5 percent in petroleum and natural gas, and less than one percent in other sectors. The second panel shows the flows of carbon tax payments on residual emissions that are made by sectors to the government cum representative agent. In all sectors the financial costs of the policy exceed the direct costs of abatement, in some cases substantially. However, whereas the latter increase monotonically with the level of the tax on emissions in all sectors, in many industries the former exhibits the expected inverted "U" shape of the Laffer curve, increasing at first but then tapering off as abatement increases and residual emissions decline. The final panel illustrates the interaction between carbon taxes and pre-existing taxes on the outputs of industry sectors. In particular, taxing carbon emissions results in significant tax shifting, inducing substantial reductions in revenues from pre-existing taxes on the output of fossil fuel sectors. Relative to the no-policy baseline, a $200/ton carbon tax displaces three-quarters of the revenue from both coal and crude petroleum, and 45 percent of that from petroleum and natural gas. However, the adverse impacts on the flows of tax revenues from much larger non-energy sectors is less severe, with payments declining by less than three percent.

Finally, CGE models' strong suit is their ability to quantify policies' economy-wide costs and macroeconomic effects in a manner that has a solid theoretical basis. On this score, the environmental and welfare consequences of carbon taxes are shown in Table 4. The model indicates that a tax of $200/ton could reduce emissions by almost two-thirds from the BaU level, which would incur a welfare cost of almost one percent of consumption. An interesting feature of the results is that the equivalent variation measure of welfare loss uniformly exceeds the reduction in GDP caused by the tax.

There are two reasons for this. The first is that the quantities of investment and net exports are held fixed, so that the influence of these components of GDP enters only through the changes in the price of commodities. Because energy commodities are a small share of GDP, the large increases in the prices of coal, petroleum and natural gas therefore have little effect. The second is the substantial revenue generated by carbon taxes—at low levels of the tax as much as four times the aggregate direct costs of abatement—which when recycled to the representative agent as lump-sum income, buoys the component of GDP corresponding to government activity. This result highlights the inaccuracy of GDP as an indicator of policies' welfare effects, as aggregate consumer surplus losses can be masked by offsetting changes in other components of national income.

8.3. Caveats to the Analysis, and Possible Remedies

It is appropriate once again to acknowledge the truth in the black-box critique. Models such as this one often have lurking within them several key driving forces that originate in their SAM data base, algebraic structure and parameter assumptions, but whose influence on the model's results remain hidden and open to misattribution. Therefore, the results generated by a highly stylized maquette such as the C-D economy should be taken with a grain of salt, as they are subject to a number of limitations that stem from the design and implementation of both the model and the experimental conditions under which it is simulated.

The first limitation is the constancy of the economy's net export position of the economy and its level of investment, discussed above. A more realistic model would permit both of these variables to adjust, the former in response to the joint effects of changes in aggregate income and the gross-of-carbon-tax domestic prices relative to world prices, and the latter due to the forward-looking behavior of households and the adjustment of saving and investment behavior to a tax shock.[24] However, since the SAM only records net exports, which are only 3 percent of GDP, the impact of terms-of-trade effects is unlikely to by significant unless exports and imports can be disaggregated into separate, price-responsive components of final demand. The model can then be re-cast in the small open economy format (e.g., Harrison et al 1997), with imports and exports linked by a balance-of-payments constraint, and commodity inputs to production or final use as an Armington (1969) composites of imported and domestically-produced varieties.

A second important shortcoming is the model's neglect of the "putty-clay" nature of capital. Jacoby and Sue Wing (1998) demonstrate the importance of capital rigidity in determining the short-run costs of the U.S. economy's adjustment to GHG emission constraints. Yet in the present analysis production is modeled as being completely reversible, and capital is modeled as a homogeneous, mobile factor whose input may be frictionlessly reallocated among producers as relative prices change. In reality, reductions in activity of the kind in Table 2 would likely entail substantial capital scrappage and associated short-run costs. The analysis can

[24] In static models, the assumption of a steady-state capital stock is a common device for specifying the demand for commodities as an input to investment as a final demand activity (e.g., Rutherford and Paltsev 1999). The evolution of the capital stock is governed by the standard perpetual inventory equation $KS' = \overline{G}_I + (1-\delta)KS$, where KS and KS' are the magnitudes of the economy's aggregate capital stock in the current and succeeding time-periods, $\overline{G}_I$ is the current quantity of aggregate investment, and δ is the rate of depreciation. If the economy is in the steady state, with capital growing at the rate ω, then $\overline{G}_I = (\omega+\delta)KS$. Additionally, the current-period aggregate return to capital is $\overline{V}_K = (r+\delta)KS$, where r is the current rate of interest. Eliminating KS by combining the preceding expressions yields the steady-state condition $\overline{G}_I = (\omega+\delta)/(r+\delta)\overline{V}_K$. Given plausible values of the parameters δ, r and ω that satisfy this relation in the SAM (e.g., assuming $\delta = 5\%$ and $\omega = 3.5\%$, the values of $\overline{V}_K$ and $\overline{G}_I$ in Figure 4 imply that $r = 9.25\%$, a good approximation of the average interest rate in 2000), the economy may be constrained to remain on the steady-state path in the presence of a shock by constraining the value of investment and capital at non-benchmark prices to maintain the steady-state relationship. This is achieved by incorporating the following side-constraint into the general equilibrium problem in (21): $\sum_{i=1}^{N}(1+\tau_i^Y)p_i s_i = (\omega+r)/(r+\delta)(1+\tau_K^F)w_K\overline{V}_K$.

therefore be significantly improved by specifying all or some of the capital input to each individual sector as a separate factor that is inelastically supplied and has its own sector-specific price. The likely consequence will be a substantial reduction in the mobility of and returns to capital—especially in declining sectors, with concomitant additional reductions in the representative agent's income and increases in the welfare costs of abatement.

A third limitation is that, like capital, labor is treated as being in inelastic supply. This, combined with the full employment assumption that is standard in many CGE models, implies that the reduction in labor demand associated with the decline in fossil fuel and energy-using sectors cannot generate unemployment. Instead, the wage falls, allowing the labor market to clear and surplus labor to move to the rest of the economy, where it is re-absorbed. But in reality labor will be far less mobile, implying that these types of price and quantity adjustments will occur more slowly, inducing frictional unemployment in the interim. This phenomenon is easily simulated by introducing a labor supply curve into the model, through which the fall in the wage attenuates the supply of labor. Depending on the relevant elasticity the distorted equilibrium may exhibit significant unemployment, but the general equilibrium interactions make it difficult to predict whether welfare will increase or decrease relative to the inelastic labor supply case.

Lastly, the model's biggest potential deficiency is the C-D assumption itself. The technologies of production and preferences in CGE models for real-world policy analysis (e.g., Bovenberg and Goulder 1996; Babiker et al 2001) are specified using nested CES production and utility functions whose substitution elasticities vary not only among levels of the nesting structure but also across sectors. To the extent that industries' production structure and input substitutability do vary in reality, the model underestimates the degree of inter-sectoral heterogeneity, implying that the results in Tables 2 and 3 and Figures 5 and 6 may suffer from a range of biases, in different directions.

Moreover, the central concern among policy makers is that mitigating CO_2 emissions will be costly because of the lack of large-scale substitutes for fossil fuels on the supply side of the economy, and the inability of producers and households to substitute non-energy inputs for fossil fuels on the demand side. In this situation the elasticities of substitution among both different fossil fuels and energy and non-energy inputs take on values that are much less than unity, with upshot that the results in Table 4 significantly underestimate carbon taxes' macroeconomic costs. The simplest way to account for this possibility is to re-cast the model as a CES economy in which the representative agent's preferences and producers' technology are CES functions, and to undertake a sensitivity analysis that compares the results of simulations with alternative combinations of values for the different elasticities. This kind of stress-testing is vital to elucidate the scope and consequences of uncertainties in CGE models' structure and assumptions.

9. Summary

This paper has sought to provide an introduction to the fundamentals of CGE modeling in a manner that is at once lucid, rigorous and practically oriented. The objective has been to de-mystify CGE models by developing a transparent, comprehensive framework within which to conceptualize their structural underpinnings, solution mechanisms and techniques of application. Beginning with the circular flow of the economy, the logic and rules of social accounting matrices were developed, and it was demonstrated how imposing the axioms of producer and consumer maximization on this framework created an algebraic model of the economy that could then be calibrated on these data. There followed a description of the techniques of model formulation, numerical calibration and solution, and a discussion of their implications for the uniqueness and stability of the simulated equilibria. In the final part of the paper the focus shifted to techniques of application, with an exposition of the use of CGE models to analyze the incidence and welfare effects of taxes, and practical demonstrations using a stylized and then a more realistic numerical example.

Despite the breadth of this survey's scope, it still does not cover many of the methodological tricks of the trade that are standard in other of areas of application of CGE models. In particular, the focus on closed economies has resulted in scant attention being paid to the important open-economy issues of macro closure rules, calibration in the presence of pre-existing tariffs, or the specification and calibration of multi-region models by combining SAMs with data on trade flows. The hope is that the framework of applied general equilibrium analysis developed in the paper provides a solid base of practical and theoretical knowledge on which the reader can build, and can thus serve as a platform for the apprehension of more advanced material on the subject across a range of different sources.

References

1. Armington, P.S. (1969). A Theory of Demand for Products Distinguished by Place of Production, IMF Staff Papers 16(1): 170-201.
2. Babiker M.H., J.M. Reilly, R.S. Eckaus, I. Sue Wing and R.C. Hyman (2001). The MIT Emissions Prediction and Policy Analysis (EPPA) Model, MIT Joint Program on the Science and Policy of Global Change Report No. 71, Cambridge MA.
3. Boehringer, C., T.F. Rutherford and W. Wiegard (2003). Computable General Equilibrium Analysis: Opening a Black Box, ZEW Discussion Paper No. 03-56, Mannheim, Germany.
4. Bovenberg, A.L. and L.H. Goulder (1996). Optimal Environmental Taxation in the Presence of Other Taxes: General- Equilibrium Analysess, American Economic Review 86(4): 985-1000.
5. Brooke, A., D. Kendrick, A. Meeraus, and R. Raman (1998). GAMS: A User's Guide, Washington DC: GAMS Development Corp.
6. Cottle, R.W., J.-S. Pang and R.E. Stone (1992). The Linear Complementarity Problem, Boston: Academic Press.
7. Dakhlia, S. (1999). Testing for a unique equilibrium in applied general equilibrium models, Journal of Economic Dynamics and Control 23: 1281-1297.
8. Dawkins, C., T.N. Srinivasan and J. Whalley (2001). Calibration, in J.J. Heckman and E. Leamer (eds.), Handbook of Econometrics Vol. 5, Amsterdam: Elsevier Science, 3653-3703.
9. Denny, K., A. Hannan and K. O'Rourke (1997). Fiscal increasing returns, unemployment and multiple equilibria: evidence from a computable general equilibrium model, mimeo, University College Dublin, Ireland.
10. Devarajan, S., D.S. Go, J.D. Lewis, S. Robinson, and P. Sinko (1997). Simple General Equilibrium Modeling, in J.F. François and K. Reinert (eds.), Applied Methods for Trade Policy Modeling, Cambridge: Cambridge University Press, 156-158.
11. Dirkse, S.P. and M.C. Ferris (1995). The PATH Solver: A Non-Monotone Stabilization Scheme for Mixed Complementarity Problems, Optimization Methods and Software 5: 123-156.
12. Ferris, M.C. and C. Kanzow (2002). Complementarity and Related Problems, in P.M. Pardalos and M.G.C. Resende (eds.), Handbook of Applied Optimization, New York: Oxford University Press, 514-530.

13. Ferris, M.C., T.S. Munson, and D. Ralph. (2000). A homotopy method for mixed complementarity problems based on the PATH solver, in D.F. Griffiths and G.A. Watson (eds.), Numerical Analysis 1999, London: Chapman and Hall: 143-167.
14. Ferris, M.C. and J.S. Pang (1997). Engineering and Economic Applications of Complementarity Problems, SIAM Review 39(4): 669-713.
15. François, J.F. (2001). General Equilibrium Studies of Multilateral Trade Negotiations: Do They Really Help?, paper presented at the Murphy Institute Conference on the Political Economy of Policy Reform, Tulane University, 9-10 November.
16. Fullerton, D., A.T. King, J.B. Shoven and J. Whalley (1981). Corporate tax integration in the United States: a general equilibrium approach, American Economic Review 71: 677-691.
17. Ginsburgh, V. and M. Keyzer (1997). The Structure of Applied General Equilibrium Models, Cambridge MA: MIT Press.
18. Goulder, L., ed. (2002). Environmental Policy Making in Economies With Prior Tax Distortions, Northampton MA: Edward Elgar.
19. Gunning, J.W. and M. Keyzer (1995). Applied General Equilibrium Models for Policy Analysis, in J. Behrman and T.N. Srinivasan (eds.) Handbook of Development Economics Vol. III-A, Amsterdam: Elsevier, 2025-2107.
20. Harrison, G.W., T.F. Rutherford and D.G. Tarr (1997). Quantifying the Uruguay Round, Economic Journal 107: 1405-1430.
21. Humphrey, T.M. (1997). Algebraic Production Functions and Their Uses Before Cobb-Douglas, Federal Reserve Bank of Richmond Economic Quarterly 83(1): 51-83.
22. Jorgenson, D.W. (1984). Econometric Methods for Applied General Equilibrium Analysis, in H. Scarf and J. Shoven (eds.), Applied General Equilibrium Analysis, Cambridge: Cambridge University Press, 139-207.
23. Kehoe, T.J. (1985). The Comparative Statics Properties of Tax Models, Canadian Journal of Economics 18: 314-34.
24. Kehoe, T.J. (1991). Computation and Multiplicity of Equilibria, in W. Hildenbrand and H. Sonnenschein (eds.), Handbook of Mathematical Economics Vol. IV, Amsterdam: North-Holland, 2049-2143.
25. Kehoe, T.J. (1998a). Social Accounting Matrices and Applied General Equilibrium Models, in I. Begg and S.G.B. Henry (eds.), Applied Economics and Public Policy, Cambridge: Cambridge University Press, 59-87.
26. Kehoe, T.J. (1998b). Uniqueness and Stability, in A.P. Kirman (ed.), Elements of General Equilibrium Analysis, Basil Blackwell, 38-87
27. Kehoe, P.J. and T.J. Kehoe (1995). A Primer on Static Applied General Equilibrium Models, in P.J. Kehoe and T.J. Kehoe (eds.), Modeling North American Economic Integration, Boston: Kluwer Academic Publishers, 1-31.
28. Kehoe, T.J. and J. Serra-Puche (1983). A Computational General Equilibrium Model with Endogenous Unemployment: An Analysis of the 1980 Fiscal Reform in Mexico, Journal of Public Economics 22: 1-26.
29. Kehoe, T.J. and J. Whalley (1985). Uniqueness of Equilibrium in Large-Scale Numerical General Equilibrium Models, Journal of Public Economics 28: 247-254.
30. King, B. (1985). What is a SAM? in J.G. Pyatt and J.I. Round (eds.), Social Accounting Matrices: A Basis for Planning, Washington D.C.: World Bank, 1-15.

31. Lofgren, H., R.L. Harris and S. Robinson (2002). A Standard Computable General Equilibrium (CGE) Model in GAMS, Microcomputers in Policy Research Working Paper No. 5, International Food Policy Research Institute, Washington DC.
32. Mas-Colell, A. (1991). On the uniqueness of equilibrium once again, in W.A. Barnett et al (eds.), Equilibrium Theory and Applications: Proceedings of the Sixth International Symposium in Economic Theory and Econometrics, New York: Cambridge University Press, 275-296.
33. Mercenier, J. (1995). Nonuniqueness of Solutions in Applied General Equilibrium Models with Scale Economies and Imperfect Competition, Economic Theory 6: 161-177.
34. Mansur, A. and J. Whalley (1983). Numerical Specification of Applied General Equilibrium Models: Estimation, Calibration and Data, in H. Scarf and J. Shoven (eds.), Applied General Equilibrium Analysis, Cambridge: Cambridge University Press, 69-127.
35. Martin, W. and L.A. Winters, eds. (1996). The Uruguay Round and the Developing Economies, New York: Cambridge University Press.
36. Shields, C.R. and J.F. François, eds. (1994). Modeling Trade Policy: Applied General Equilibrium Assessments of North American Free Trade, New York: Cambridge University Press.
37. Paltsev, S. (2004). Moving from Static to Dynamic General Equilibrium Economic Models: Notes for a beginner in MPSGE, mimeo, Massachusetts Institute of Technology, Cambridge MA.
38. Panagariya, A. and R. Duttagupta (2001). The "Gains" from Preferential Trade Liberalization in the CGE Models: Where do they Come From? in S. Lahiri (ed.), Regionalism and Globalization: Theory and Practice, London and New York: Routledge: 39-60.
39. Perry, G., J. Whalley and G. McMahon, eds. (2001). Fiscal Reform and Structural Change in Developing Countries, New-York: Palgrave-Macmillan.
40. Reinert, K.A. and D.W. Roland-Holst (1992). A Detailed Social Accounting Matrix for the USA, 1988, Economic Systems Research 4(2): 173-187.
41. Rutherford, T.F. (1995). Extensions of GAMS for Complementarity Problems Arising in Applied Economic Analysis, Journal of Economic Dynamics and Control 19(8): 1299-1324.
42. Rutherford, T.F. (1999). Applied General Equilibrium Modeling with MPSGE as a GAMS Subsystem: An Overview of the Modeling Framework and Syntax, Computational Economics, 14: 1-46.
43. Rutherford, T.F. and S. Paltsev (1999). From an Input-Output Table to a General Equilibrium Model: Assessing the Excess Burden of Indirect Taxes in Russia, mimeo, University of Colorado, Boulder CO.
44. Samuelson, P.A. (1979). Paul Douglas's Measurement of Production Functions and Marginal Productivities, Journal of Political Economy 87: 923-39.
45. Sandelin, B. (1976). On the Origin of the Cobb-Douglas Production Function, Economy and History 19 (2): 117-23.
46. Shoven, J.B. and J.L. Whalley (1984). Applied General Equilibrium Models of Taxation and International Trade: An Introduction and Survey, Journal of Economic Literature 22: 1007-1051.
47. Shoven, J.B. and J.L. Whalley (1992). Applying General Equilibrium, Cambridge: Cambridge University Press.
48. U.S. Dept. of Energy/Energy Information Administration (2003). Emissions of Greenhouse Gases in the United States 2002, Report No. DOE/EIA-0573(2002), Washington D.C.

49. Varian, H.R. (1992). Microeconomic Analysis, New York: Norton.
50. Weyant, J., ed. (1999). The Costs of the Kyoto Protocol: a Multi-Model Evaluation, Energy Journal special issue.
51. Whalley, J. and S. Zhang (2002). Tax Induced Multiple Equilibria, mimeo, University of Western Ontario, Ontario, Canada.

Figure 1. The Circular Flow of the Economy

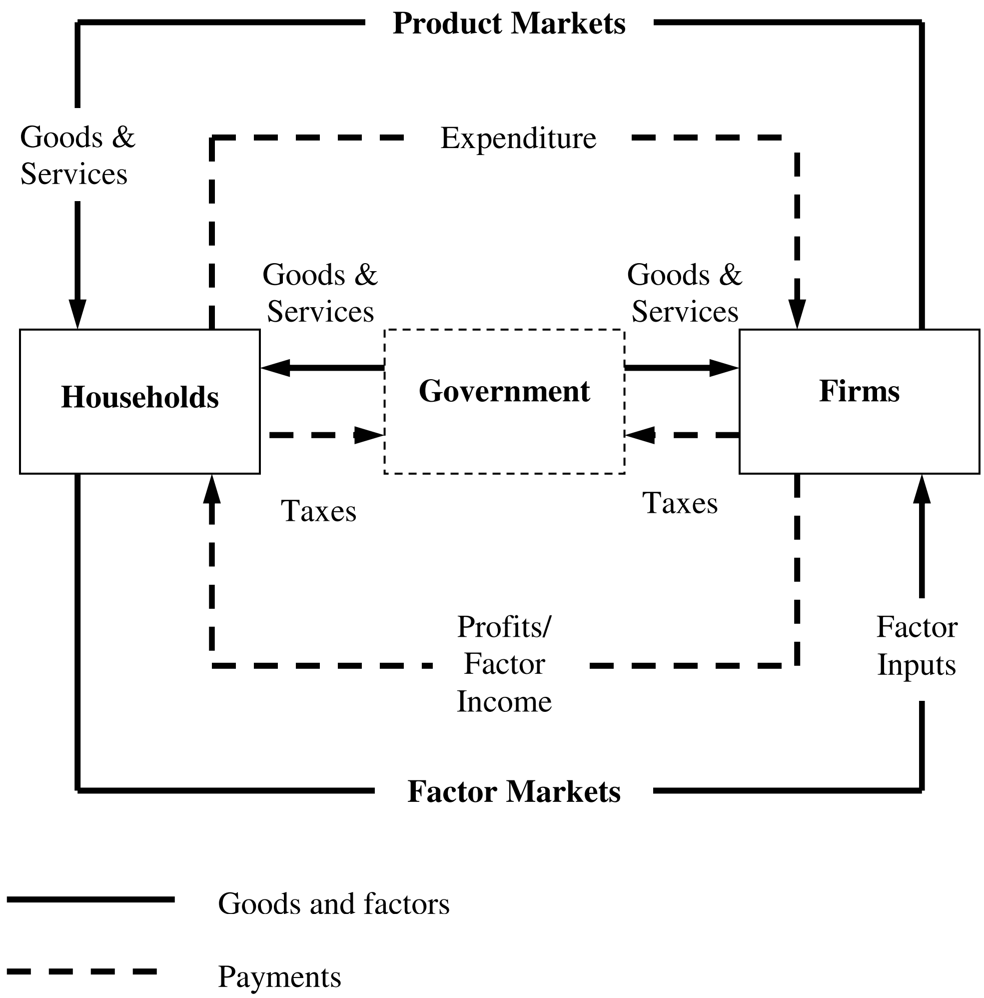
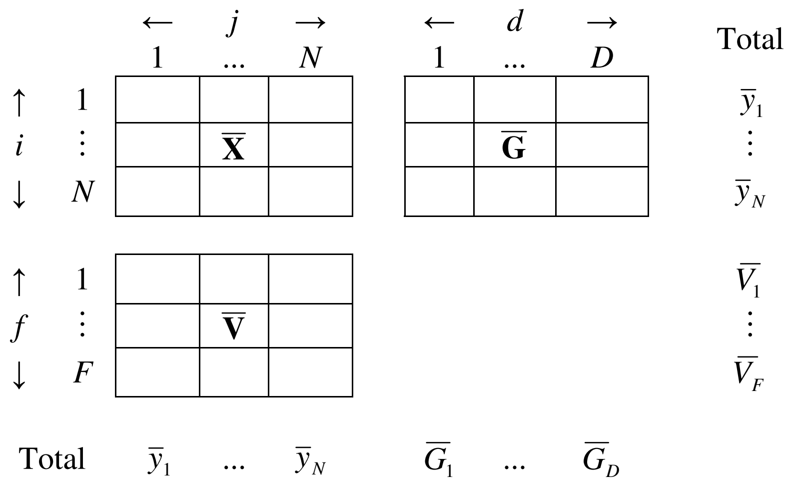
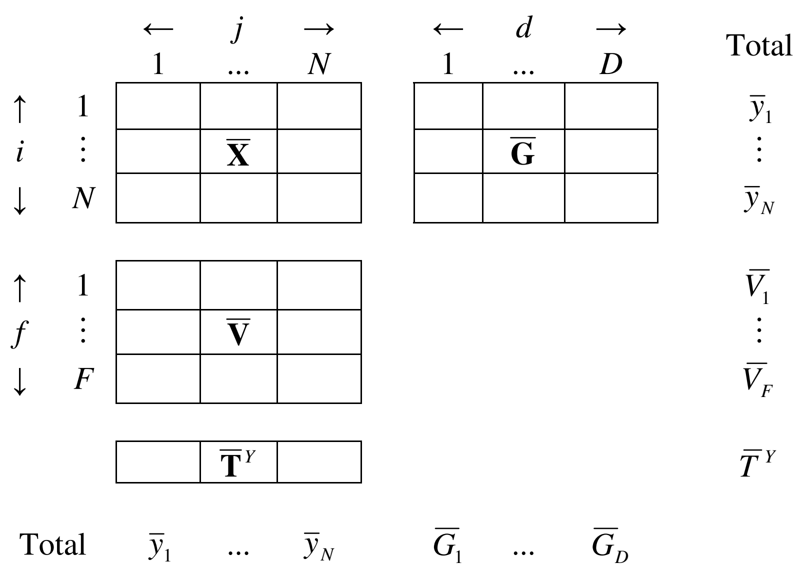


Figure 2. A Stylized Social Accounting Matrix



(a) No distortions



(b) Taxes on industry outputs

Figure 3. A SAM for the 2 × 2 × 1 Economy

	1	2	C	S	Total
1	10	30	50	30	120
2	20	10	60	10	100
L	30	50			80
K	60	10			70
Total	120	100	110	40	

Figure 4. Year 2000 Social Accounting Matrix for the U.S. (2000 Dollars × 10^{10})

	Coal Mining	Electric Power	Natural Gas	Crude Oil & Gas	Refined Petroleum	Energy Intensive Mfg.	Transport-ation	Rest of the Economy	Cons-umption	Invest-ment	Net Exports	Total
Coal Mining	0.243	1.448	0.004	0.000	0.001	0.219	0.013	0.238	0.014	0.000	0.108	2.288
Electric Power	0.052	0.084	0.027	0.118	0.168	1.384	0.283	9.530	12.915	0.000	-0.093	24.467
Gas	0.003	0.526	2.283	0.446	0.246	0.817	0.056	2.199	4.136	0.000	0.045	10.757
Crude Oil & Gas	0.000	0.024	4.795	2.675	8.381	0.939	0.030	0.120	0.013	0.072	-6.189	10.861
Refining	0.066	0.238	0.038	0.072	1.753	0.628	2.428	4.950	8.345	0.128	-0.542	18.105
Energy Intensive Mfg.	0.101	0.121	0.015	0.285	0.513	17.434	0.177	47.534	9.239	0.906	-3.506	72.820
Transport-ation	0.158	0.945	0.135	0.122	0.784	3.548	9.796	19.835	17.316	1.492	5.107	59.238
Rest of the Economy	0.747	5.142	1.897	4.694	2.798	19.974	16.055	540.977	751.254	203.063	-21.406	1525.196
Labor	0.437	4.422	0.434	0.665	1.141	16.128	19.032	553.948				596.208
Capital	0.278	8.830	0.866	1.525	2.115	10.806	9.792	310.641				344.852
Taxes	0.203	2.686	0.263	0.258	0.204	0.944	1.574	35.225				41.358
Total	2.288	24.467	10.757	10.861	18.105	72.820	59.238	1525.196	803.233	205.660	-26.465	

Value added = GDP = 9.82 Trillion dollars
Gross Output = 17.24 Trillion dollars

Source: Bureau of Economic Analysis data files; author's calculations and assumptions.

Figure 5. Impact of Carbon Taxes on Carbon Emissions by Sector

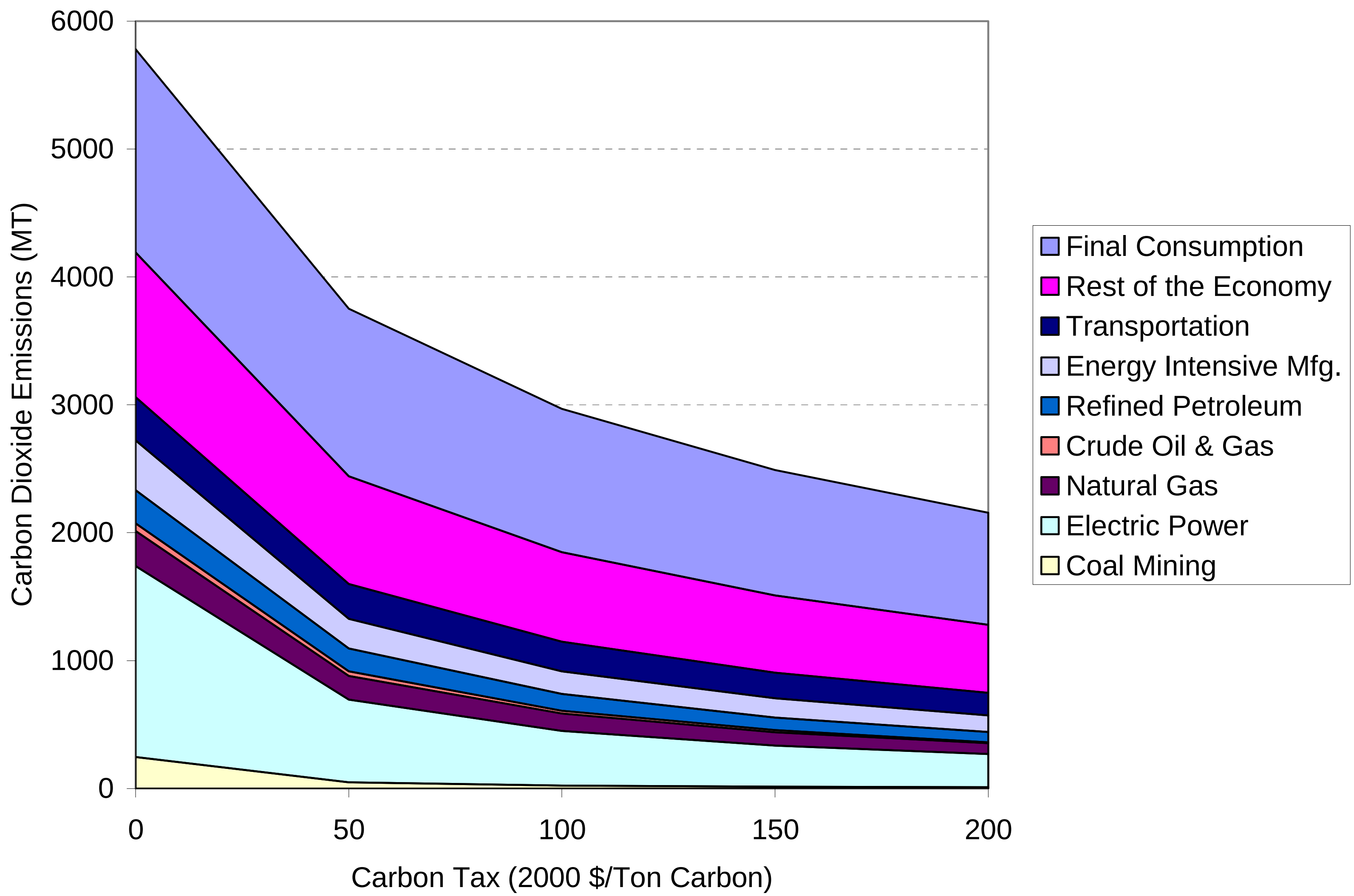


Figure 6. Year 2000 Sectoral Marginal Abatement Cost Curves for the U.S.

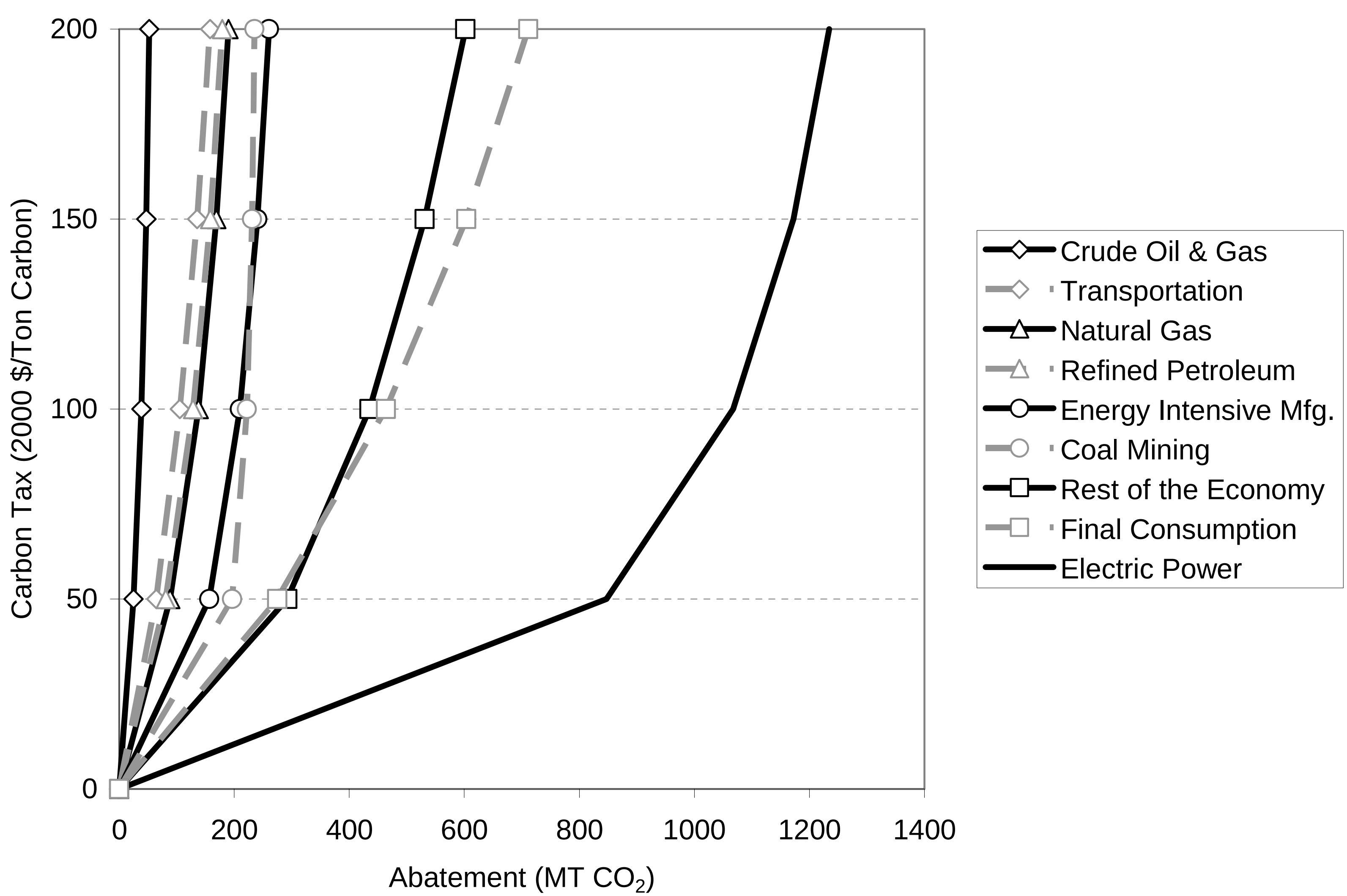


Table 1. The Effects of Taxes on the 2 × 2 × 1 Economy

Variable	Benchmark	Effect of 50 percent tax imposed on					
		Output		Consumption		Factors	
		y_1	y_2	c_1	c_2	V_L	V_K
p_1	1	1.3	0.7	0.8	0.8	1	1
p_2	1	0.8	1.3	0.8	0.8	1	1
y_1	120	87.6	120.4	112.9	127.4	120	120
y_2	100	98.4	65.2	109.3	90.3	100	100
x_{11}	10	3.7	10.0	9.4	10.6	10	10
x_{12}	30	18.7	17.9	33.6	26.4	30	30
x_{21}	20	11.5	11.0	18.4	21.8	20	20
x_{22}	10	9.8	3.3	10.9	9.0	10	10
c_1	50	35.3	62.5	39.9	60.3	50	50
c_2	60	67.0	41.0	70.0	49.5	60	60
w_L	1	0.7	0.5	0.9	0.8	0.7	1
w_K	1	0.5	0.7	0.8	0.8	1	0.7
v_{L1}	30	20.8	40.2	26.8	33.6	30	30
v_{L2}	50	59.2	39.8	53.2	46.4	50	50
v_{K1}	60	54.5	63.7	58.4	61.5	60	60
v_{K2}	10	15.5	6.3	11.6	8.5	10	10
$m - s$	110	99.7	98.9	108.0	107.9	110	110.0
EV (%)	–	-9.3	-10.1	-1.9	-1.9	0.0	0.0

Variable	Benchmark	Effect of 50 percent tax imposed on							
		Intermediate inputs by sector				Factor inputs by sector			
		x_{11}	x_{12}	x_{21}	x_{22}	v_{L1}	v_{L2}	v_{K1}	v_{K2}
p_1	1	1.0	0.9	1.0	1.0	1.1	0.9	1.0	1.0
p_2	1	1.0	1.1	1.0	1.0	0.9	1.1	1.0	1.0
y_1	120	114.6	113.0	113.6	119.9	113.4	125.4	116.6	122.2
y_2	100	99.7	91.1	93.7	94.1	104.3	93.5	102.1	97.2
x_{11}	10	6.4	9.4	9.5	10.0	9.5	10.4	9.7	10.2
x_{12}	30	29.2	20.8	26.4	29.3	27.7	31.9	28.8	30.8
x_{21}	20	19.6	16.5	13.4	19.3	21.3	18.3	20.6	19.3
x_{22}	10	10.0	9.1	9.4	6.3	10.4	9.3	10.2	9.7
c_1	50	49.0	52.8	47.7	50.6	46.2	53.0	48.0	51.2
c_2	60	60.2	55.5	60.9	58.6	62.6	55.8	61.3	58.2
w_L	1	1.0	0.9	0.9	1.0	0.9	0.8	1.0	1.0
w_K	1	1.0	0.9	1.0	1.0	1.0	1.0	0.7	0.9
v_{L1}	30	29.6	28.2	31.4	30.4	23.2	37.5	30.2	29.8
v_{L2}	50	50.4	51.8	48.6	49.6	56.8	42.5	49.8	50.2
v_{K1}	60	59.8	59.1	60.6	60.2	60.2	59.8	56.1	62.9
v_{K2}	10	10.2	10.9	9.4	9.8	9.8	10.2	13.9	7.1
$m - s$	110	109.2	108.1	108.6	109.2	108.6	108.5	109.2	109.4
EV (%)	–	-0.7	-1.8	-1.3	-0.7	-1.3	-1.4	-0.7	-0.6

Table 2. The Sectoral Impacts of Carbon Taxes on the U.S. Economy

Carbon Tax (2000 $/Ton)	Coal Mining	Electric Power	Natural Gas	Crude Oil & Gas	Refining	Energy Intensive Mfg.	Transport-ation	Rest of the Economy
Changes in Gross-of-Tax Commodity Prices (percent)								
50	143.5	6.2	20.3	0.8	20.2	0.5	0.4	-0.4
100	281.9	9.6	39.9	1.6	40.0	0.8	0.8	-0.7
150	418.6	12.0	59.1	2.2	59.6	1.1	1.2	-1.0
200	554.4	13.8	77.9	2.7	79.0	1.3	1.5	-1.2
Changes in Final Consumption by Commodity (percent)								
50	-59.0	-6.0	-17.0	-1.0	-17.0	-0.7	-0.6	0.2
100	-73.9	-9.1	-28.8	-2.0	-28.9	-1.2	-1.2	0.3
150	-80.9	-11.3	-37.6	-2.8	-37.8	-1.7	-1.8	0.4
200	-84.9	-12.9	-44.3	-3.5	-44.6	-2.1	-2.3	0.4
Changes in Demand for Coal by Sector (percent)								
50	-81.2	-59.3	-66.4	–	-66.0	-59.3	-59.3	-59.2
100	-91.5	-74.2	-81.7	–	-81.5	-74.2	-74.2	-74.1
150	-95.0	-81.1	-88.1	–	-88.0	-81.1	-81.1	-81.0
200	-96.6	-85.1	-91.6	–	-91.6	-85.1	-85.1	-85.0
Changes in Demand for Petroleum by Sector (percent)								
50	-61.8	-17.5	-31.9	-41.0	-31.2	-17.6	-17.5	-17.3
100	-76.9	-29.6	-50.0	-63.4	-49.4	-29.7	-29.6	-29.3
150	-83.7	-38.5	-61.4	-77.0	-61.1	-38.7	-38.5	-38.2
200	-87.5	-45.4	-69.2	-85.7	-69.1	-45.6	-45.4	-45.0
Changes in Demand for Natural Gas by Sector (percent)								
50	-61.8	-17.5	-31.9	-41.0	-31.3	-17.6	-17.6	-17.3
100	-76.9	-29.5	-49.9	-63.4	-49.4	-29.7	-29.6	-29.2
150	-83.6	-38.3	-61.3	-76.9	-61.0	-38.5	-38.3	-38.0
200	-87.5	-45.0	-69.0	-85.6	-68.9	-45.2	-45.1	-44.7
Changes in Demand for Electricity by Sector (percent)								
50	-56.8	-6.6	-22.9	-33.2	-22.1	-6.7	-6.7	-6.3
100	-70.5	-10.0	-36.1	-53.3	-35.4	-10.2	-10.1	-9.6
150	-76.8	-12.3	-45.0	-67.2	-44.6	-12.6	-12.4	-11.9
200	-80.4	-14.1	-51.5	-77.5	-51.4	-14.4	-14.1	-13.6
Changes in Sectoral Activity Levels (percent)								
50	-58.8	-6.6	-21.6	-29.7	-19.2	-1.4	-1.3	-0.1
100	-72.5	-10.0	-35.3	-49.6	-32.1	-2.4	-2.2	-0.2
150	-78.7	-12.3	-44.8	-64.0	-41.5	-3.1	-3.0	-0.3
200	-82.3	-14.1	-51.8	-75.1	-48.7	-3.8	-3.7	-0.4

Table 3. The Incidence of Carbon Taxes and Abatement Costs

Carbon Tax (2000 $/Ton)	Coal Mining	Electric Power	Natural Gas	Crude Oil & Gas	Refining	Energy Intensive Mfg.	Transport-ation	Rest of the Economy	Final Cons-umption
Carbon Tax Payments by Sector (2000 billion $)									
50	3.3	20.4	3.7	0.8	3.5	5.3	4.6	15.5	21.6
100	1.3	17.7	5.1	1.0	4.8	6.3	7.4	22.9	35.7
150	0.9	17.5	5.6	0.9	5.3	7.3	9.5	28.6	45.9
200	0.7	17.6	5.7	0.8	5.5	8.1	11.0	32.9	53.6
Direct Abatement Costs by Sector (2000 billion $)									
50	1.3	5.8	0.6	0.2	0.6	1.1	0.4	2.0	1.9
100	1.9	10.3	1.6	0.5	1.5	2.2	1.3	4.9	5.7
150	2.2	13.9	2.7	0.7	2.5	3.2	2.3	8.2	10.5
200	2.4	16.8	3.7	1.0	3.5	4.2	3.4	11.6	15.6
Change in Output Tax Payments by Sector from Benchmark (percent)									
50	-54.1	-0.8	-18.1	-29.1	-17.3	-0.9	-0.9	-0.5	–
100	-67.7	-1.4	-30.0	-48.8	-29.2	-1.6	-1.5	-1.0	–
150	-74.0	-1.8	-38.4	-63.3	-38.0	-2.1	-1.9	-1.3	–
200	-77.7	-2.2	-44.8	-74.4	-44.7	-2.6	-2.3	-1.6	–

Table 4. The Aggregate Economic Impacts of Carbon Taxes

Carbon Tax (2000 $/Ton)	Emissions (MT)	Abate-ment (% of BaU)	GDP (2000 trillion $)	GDP Change from BaU (%)	Carbon Tax Payments (2000 billion $)	Share of GDP (%)	Direct Abatement Costs (2000 billion $)	Share of GDP (%)	Cons-umption (2000 trillion $)	Equiv-alent Variation (%)
0	5796	–	9.82	–	–	–	–	–	8.03	–
50	3768	65	9.80	-0.08	51.4	5.2	13.8	1.4	8.02	-0.2
100	2986	52	9.78	-0.14	81.4	8.3	29.8	3.0	8.00	-0.43
150	2507	43	9.75	-0.19	102.6	10.5	46.1	4.7	7.98	-0.66
200	2172	37	9.73	-0.23	118.5	12.2	62.1	6.4	7.96	-0.88

Appendix A: Solving the Representative Agent's Problem

The problem is solved by forming the lagrangian for the representative agent's utility production:

$$\mathcal{L}^C = p_U U - \sum_{i=1}^{N} p_i c_i + \lambda^C \left(U - A_C \prod_{i=1}^{N} c_i^{\alpha_i} \right)$$

and finding the first-order conditions by taking the derivative with respect to the consumer's consumption of each good:

$$\text{(A-1)} \quad \frac{\partial \mathcal{L}^C}{\partial c_i} = -p_i - \lambda^C \frac{\alpha_i}{c_i} A_C \prod_{i=1}^{N} c_i^{\alpha_i} = 0 .$$

Using this equation to compare the consumption of commodities, say 1 and 2, by taking the ratio of the first-order conditions we have

$$\frac{\alpha_1 / c_1}{\alpha_2 / c_2} = \frac{p_1}{p_2} \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{p_1 c_1}{p_2 c_2} ,$$

so that the ratio of the exponents of the Cobb-Douglas utility function is equal to the ratio of the shares of the agent's expenditure of consumption. Thus, the α_is have a natural interpretation as expenditure shares, which makes sense given that the α_is sum to unity. Thus, rearranging the first order conditions in (A-1) and adding them up over all *i* commodities gives an expression for the lagrange multiplier

$$\text{(A-2)} \quad -\lambda^C A_C \prod_{i=1}^{N} c_i^{\alpha_i} \times \sum_{i=1}^{N} \alpha_i = \sum_{i=1}^{N} p_i c_i \Rightarrow \lambda^C = -\frac{1}{U} \left(m - \sum_{i=1}^{N} p_i s_i \right).$$

It is useful to also take the derivative of the Lagrangian with respect to utility, to give

$$\text{(A-3)} \quad \frac{\partial \mathcal{L}^C}{\partial U} = p_U + \lambda^C = 0 \Rightarrow \lambda^C = -p_U .$$

Together, eqs. (A-2) and (A-3) suggest that the price of utility is the average utility of income allocated to consumption:

$$p_U = \frac{1}{U} \left(m - \sum_{i=1}^{N} p_i s_i \right).$$

This is simply the price of aggregate consumption, or, equivalently, the consumer price index of the economy, whose value, when fixed at unity gives a natural numeraire by which to deflate all of the other prices in the model. Eq. (A-2) may be substituted back into (A-1) to yield eq. (9) in the paper.

Appendix B: Solving the Producer's Problem

The problem is solved by forming the lagrangian for the j^{th} producer

$$\mathcal{L}_j^P = p_j y_j - \sum_{i=1}^{N} p_i x_{ij} - \sum_{f=1}^{F} w_f v_{fj} + \lambda_j^P \left(y_j - A_j \prod_{i=1}^{N} x_{ij}^{\beta_{ij}} \prod_{f=1}^{F} v_{fj}^{\gamma_{fj}} \right)$$

and taking derivatives with respect to the producer's use of each intermediate good and primary factor to yield the first-order conditions:

$$\text{(B-1)} \quad \frac{\partial \mathcal{L}_j^P}{\partial x_{ij}} = -p_i - \lambda_j^P \frac{\beta_{ij}}{x_{ij}} A_j \prod_{i=1}^{N} x_{ij}^{\beta_{ij}} \prod_{f=1}^{F} v_{fj}^{\gamma_{fj}} = 0$$

and

$$\text{(B-2)} \quad \frac{\partial \mathcal{L}_j^P}{\partial v_{fj}} = -w_f - \lambda_j^P \frac{\gamma_{fj}}{v_{fj}} A_j \prod_{i=1}^{N} x_{ij}^{\beta_{ij}} \prod_{f=1}^{F} v_{fj}^{\gamma_{fj}} = 0.$$

It is useful to also take the derivative of the Lagrangian with respect to output, to give

$$\frac{\partial \mathcal{L}_j^P}{\partial y_j} = p_j + \lambda_j^P = 0 \Rightarrow \lambda_j^P = -p_j.$$

Substituting this result into (B-1) and (B-2) yields eqs. (11) and (12), respectively.

Appendix C: GAMS/MPSGE Code for Tax Effects in the 2 × 2 × 1 Economy

```
$title:      a 2 x 2 x 1 maquette of tax effects in general equilibrium
$stitle:     copyright 2004, Ian Sue Wing (isw@bu.edu), Boston University
$stitle:     code provided without warranty or support

table sam(*,*)      benchmark social accounting matrix

       1      2      C      S
1      10     30     50     30
2      20     10     60     10
L      30     50
K      60     10
;

sets
       i      commodities   /1, 2/
       f      factors       /l labor, k capital/
       d      demands       /c consumption, s saving/

parameters
       xbar   benchmark intermediate transactions matrix
       vbar   benchmark factor supply matrix
       gbar   benchmark final demand matrix
       ybar   benchmark output

       ty     tax on sectoral output
       tx     tax on intermediate inputs
       tv     tax on factor inputs
       tc     tax on consumption
;

alias (i,j);

xbar(i,j)     =      sam(i,j);
vbar(f,j)     =      sam(f,j);
gbar(i,d)     =      sam(i,d);
ybar(j)       =      sum(i, xbar(i,j)) + sum(f, vbar(f,j));

*      all taxes are zero in benchmark

ty(j)         =      0;
tx(i,j)       =      0;
tv(f,j)       =      0;
tc(i)         =      0;

$ontext

$model: maquette

$commodities:
       p(i)   ! price index for commodities
       w(f)   ! price index for factors
       pu     ! aggregate consumption price (numeraire)

$sectors:
       y(j)   ! producing sectors
       u      ! production of utility good (utility function)

$consumers:
       ra     ! representative agent

$report:
       v:qy(j)       o:p(j) prod:y(j)
       v:qx(i,j)     i:p(i) prod:y(j)
       v:qc(i)       i:p(i) prod:u
       v:qv(f,j)     i:w(f) prod:y(j)

$prod:y(j)           s:1
       o:p(j) q:ybar(j)    a:ra   t:ty(j)
```

```
        i:p(i) q:xbar(i,j)  a:ra   t:tx(i,j)
        i:w(f) q:vbar(f,j)  a:ra   t:tv(f,j)

$prod:u             s:1
        o:pu         q:(sum(i, gbar(i,"c")))
        i:p(i) q:gbar(i,"c") a:ra  t:tc(i)

$demand:ra
        d:pu         q:(sum(i, gbar(i,"c")))
        e:p(i) q:(-gbar(i,"s"))
        e:w(f) q:(sum(j, vbar(f,j)))

$offtext

$sysinclude mpsgeset maquette

*       set numeraire

pu.fx               =      1;

option mcp          =      path;

*       benchmark replication

maquette.iterlim    =      0;
$include maquette.gen
solve maquette using mcp;

*       free solve

maquette.iterlim    =      8000;
$include maquette.gen
solve maquette using mcp;

*       suspend listing to save memory

$offlisting

table  taxpol(*,*,*) matrix of tax policy cases

        y.1    y.2    c.1    c.2    1.1    1.2    2.1    2.2    L.1    L.2    K.1    K.2
i1      0      0      0      0      0      0      0      0      0      0      0      0
i2      0.5
i3      0      0.5
i4      0      0      0.5
i5      0      0      0      0.5
i6      0      0      0      0      0      0      0      0      0.5    0.5
i7      0      0      0      0      0      0      0      0      0      0      0.5    0.5
i8      0      0      0      0      0.5
i9      0      0      0      0      0      0.5
i10     0      0      0      0      0      0      0.5
i11     0      0      0      0      0      0      0      0.5
i12     0      0      0      0      0      0      0      0      0.5
i13     0      0      0      0      0      0      0      0      0      0.5
i14     0      0      0      0      0      0      0      0      0      0      0.5
i15     0      0      0      0      0      0      0      0      0      0      0      0.5
;

sets
        iter         iterations   /i1*i15/

parameters
        ra0          benchmark income of representative agent
        results      array to hold results
;

*       record the value of benchmark income

ra0    =      ra.l;

loop(iter,
```

```
*      first always solve a benchmark case in which all taxes are zero

       ty(j)         =      0;
       tx(i,j)       =      0;
       tv(f,j)       =      0;
       tc(i)         =      0;

$include maquette.gen
       solve maquette using mcp;

*      now solve for the different tariff-ridden equilibria

       ty(j)         =      taxpol(iter,"y",j);
       tx(i,j)       =      taxpol(iter,i,j);
       tv(f,j)       =      taxpol(iter,f,j);
       tc(i)         =      taxpol(iter,"c",i);

$include maquette.gen
       solve maquette using mcp;

       results("p_1",iter) =      p.l("1");
       results("p_2",iter) =      p.l("2");
       results("y_1",iter) =      qy.l("1");
       results("y_2",iter) =      qy.l("2");
       results("x_1_1",iter)      =      qx.l("1","1");
       results("x_1_2",iter)      =      qx.l("1","2");
       results("x_2_1",iter)      =      qx.l("2","1");
       results("x_2_2",iter)      =      qx.l("2","2");
       results("c_1",iter) =      qc.l("1");
       results("c_2",iter) =      qc.l("2");
       results("w_l",iter) =      w.l("l");
       results("w_k",iter) =      w.l("k");
       results("v_l_1",iter)      =      qv.l("l","1");
       results("v_l_2",iter)      =      qv.l("l","2");
       results("v_k_1",iter)      =      qv.l("k","1");
       results("v_k_2",iter)      =      qv.l("k","2");
       results("m",iter)          =      ra.l;
       results("% ev",iter)=      100 * (ra.l / ra0 - 1);

);

display results;
```

Appendix D: GAMS/MPSGE Code for Carbon Taxes and the U.S. Economy

```
$title:      a simple static CGE model of carbon taxes in the u.s. economy
$stitle:     copyright 2004, Ian Sue Wing (isw@bu.edu), Boston University
$stitle:     code provided without warranty or support

*---------------------------------*
* set and parameter declarations *
*---------------------------------*

sets
       i      industry sectors    /
              col    coal mining
              o_g    crude oil and gas
              gas    gas works and distribution
              oil    refined petroleum
              ele    electric power
              eis    energy intensive industry sectors
              trn    transportation
              roe    the rest of the economy/

       e(i)   energy industries   /col, gas, oil, ele/
       ne(i)  non-energy industries

       f      primary factors     /
              l      labor
              k      capital/

       d      final demands /
              cons   consumption
              inv    investment
              nx     net exports/

       cd(d)  consumption demand
       id(d)  investment demand
       nd(d)  net export demand

parameters
       x0     benchmark intermediate transactions matrix (10 billion 2000 usd)
       v0     benchmark factor supply matrix (10 billion 2000 usd)
       g0     benchmark final demand matrix (10 billion 2000 usd)

       tax0   benchmark net tax revenue (10 billion 2000 usd)
       tr0    benchmark tax rate on output

       y0     benchmark aggregate output (10 billion 2000 usd)
       cons0  benchmark consumption (10 billion 2000 usd)
       inv0   benchmark investment (10 billion 2000 usd)
       nx0    benchmark net exports (10 billion 2000 usd)
       a0     benchmark armington aggregate use (10 billion 2000 usd)
;

alias (i,j);

cd(d)$sameas(d,"cons")     =      yes;
id(d)$sameas(d,"inv")      =      yes;
nd(d)$sameas(d,"nx")=      yes;
ne(i)$(not e(i))           =      yes;

*-------------------------------------*
* aggregate social accounting matrix *
*-------------------------------------*

table sam(*,*)      2000 social accounting matrix for usa (10 billion 2000 usd --
constructed from bea 2002 make and use tables employing the industry-technology
assumption)

    col   ele   gas   o_g   oil   eis    trn    roe     cons    inv    nx
col 0.243 1.448 0.004 0.000 0.001 0.219  0.013  0.238   0.014   0.000  0.108
ele 0.052 0.084 0.027 0.118 0.168 1.384  0.283  9.530   12.915  0.000  -0.093
```

```
gas 0.003 0.526 2.283 0.446 0.246 0.817  0.056  2.199   4.136   0.000  0.045
o_g 0.000 0.024 4.795 2.675 8.381 0.939  0.030  0.120   0.013   0.072  -6.189
oil 0.066 0.238 0.038 0.072 1.753 0.628  2.428  4.950   8.345   0.128  -0.542
eis 0.101 0.121 0.015 0.285 0.513 17.434 0.177  47.534  9.239   0.906  -3.506
trn 0.158 0.945 0.135 0.122 0.784 3.548  9.796  19.835  17.316  1.492  5.107
roe 0.747 5.142 1.897 4.694 2.798 19.974 16.055 540.977 751.254 203.063 -21.41
l   0.437 4.422 0.434 0.665 1.141 16.128 19.032 553.948
k   0.278 8.830 0.866 1.525 2.115 10.806 9.792  310.641
tax 0.203 2.686 0.263 0.258 0.204 0.944  1.574  35.225
;

*-----------------------*
* benchmark calibration *
*-----------------------*

*       extract benchmark matrices

x0(i,j)         =       sam(i,j);
v0(f,j)         =       sam(f,j);
g0(i,d)         =       sam(i,d);

*       extract distortions

tax0(j)         =       sam("tax",j);

*       transfer tax and subsidy revenue into tax rates on output

tr0(j) =        tax0(j) / (sum(i,x0(i,j)) + sum(f,v0(f,j)) + tax0(j));

*       useful aggregates

y0(j)           =       sum(i,x0(i,j)) + sum(f,v0(f,j)) + tax0(j);
nx0(i) =        sum(nd,g0(i,nd));
cons0           =       sum((i,cd),g0(i,cd));
inv0            =       sum((i,id),g0(i,id));
a0(i)           =       y0(i) - nx0(i);

display v0, g0, tr0, y0, nx0, a0;

*-------------------------------*
* energy and emissions accounts *
*-------------------------------*

parameters
        co2(e) co2 emissions by fuel in 2000 (mt -- from eia 2003)   /
                col     2112
                oil     2439.4
                gas     1244.3/
        ccoef           co2 coefficient on energy (tons of co2 per dollar)

scalars
        co2_carb        co2 to carbon molecular weight conversion factor
        carblim0        benchmark carbon emission rights
        carblim         carbon emission rights     /0/
        carbtax         carbon tax                 /0/
        ra0             benchmark income level of representative agent
;

co2_carb        =       12 / 44;

*       multiply by 1e-4 to convert co2 in mt (1e6) to 10 billion ton (1e10)
*       then, a0 in 10 billion $ (1e10) implies ccoef in tons/$
*       and carbon price in $/ton

ccoef(e)        =       1e-4 * co2(e) / a0(e);

display ccoef;

carblim0        =       1e-4 * sum(e, co2(e));

*------------*
* core model *
```

```
*------------*

$ontext

$model: usa_co2

$sectors:
	y(j)		! production by industries
	cons		! consumption
	carbon(e)	! dummy aggregate carbon accounting sector

$commodities:
	p(i)		! price index of commodities
	w(f)		! price index of primary factors
	pcons		! price index of aggregate consumption
	pce(e)		! gross-of-carbon-tax energy price
	pcarb$carblim	! carbon tax dual to quantitative emission limit

$consumers:
	ra		! representative agent

$auxiliary:
	ctax$carbtax	! tax on aggregate carbon emissions

$report:
	v:qcarb(e)$carblim	i:pcarb		prod:carbon(e)		! co2 by fuel
	v:qd(i)			o:p(i)	prod:y(i)	! domestic output
	v:necons(i)$ne(i)	i:p(i)	prod:cons	! non-energy goods consumed
	v:econs(e)		i:pce(e)	prod:cons	! energy goods consumed
	v:qeint(e,j)	i:pce(e)	prod:y(j)	! sectoral energy inputs

*	production for domestic use and export

$prod:y(j)		s:1
	o:p(j)	q:y0(j)		p:(1 + tr0(j))		a:ra	t:tr0(j)
	i:p(ne)		q:x0(ne,j)
	i:pce(e)	q:x0(e,j)
	i:w(f)	q:v0(f,j)

*	final demand aggregation: consumption

$prod:cons		s:1
	o:pcons		q:cons0
	i:p(ne)		q:(sum(cd,g0(ne,cd)))
	i:pce(e)	q:(sum(cd,g0(e,cd)))

*	emission accounting

$prod:carbon(e)		s:0
	o:pce(e)	q:a0(e)
	i:p(e)	q:a0(e)
	i:pcarb$carblim		q:(ccoef(e) * a0(e))

*	income, demands, and endowments of representative agent

$demand:ra

*	aggregate consumption

	d:pcons		q:cons0

*	factor endowment

	e:w(f)	q:(sum(j,v0(f,j)))

*	investment aggregate demands (model as negative endowments)

	e:p(ne)		q:(sum(id,-g0(ne,id)))
	e:pce(e)	q:(sum(id,-g0(e,id)))

*	net exports (model as exogenous endowment at domestic prices)
```

```
        e:p(i) q:(-nx0(i))

*       emission permit endowment

        e:pcarb$carblim     q:carblim    r:ctax$carbtax

*        emission tax dual to permit endowment

$constraint:ctax$carbtax
        pcarb  =e=    carbtax;

$offtext
$sysinclude mpsgeset usa_co2
option mcp          =      path;

*-----------------------*
* benchmark replication *
*-----------------------*

ctax.l       =      0;
usa_co2.iterlim     =      0;
$include usa_co2.gen
solve usa_co2 using mcp;

*       set consumption price index as numeraire

pcons.fx            =      1;

*       free solve

usa_co2.iterlim     =      8000;
$include usa_co2.gen
solve usa_co2 using mcp;

ra0                 =      ra.l;

*       impose emission limits

*       check that benchmark emissions are a non-binding constraint on economy:
*       level value of variable pcarb should be zero at solution point
*       prices should remain at unity and quantities should replicate benchmark

carblim             =      carblim0;
pcarb.l             =      1;

$include usa_co2.gen
solve usa_co2 using mcp;

*       now suppress listing to save memory

$offlisting
$offsymxref offsymlist

options
        limrow =      0
        limcol =      0
        solprint      =      off
        sysout =      off
;

*-----------------*
* policy analysis *
*-----------------*

sets
        iter          iteration over level of carbon constraint /iter1 * iter5/
;

parameters
        results             array for reporting aggregate results
        p_impacts           price impacts (percent change)
        q_impacts           quantity impacts (percent change)
```

```
        c_impacts           consumption impacts (percent change)
        coal_impacts  coal input impacts by sector (percent change)
        oil_impacts         oil input impacts by sector (percent change)
        gas_impacts         gas input impacts by sector (percent change)
        elec_impacts  electricity input impacts by sector (percent change)
        emiss               sectoral co2 emissions (mt)
        mac                 sectoral marginal abatement cost curves
;

loop(iter,

*       perform benchmark solve first before computing distorted equilibrium

        carblim             =       carblim0;
        carbtax             =       0;

$include usa_co2.gen
        solve usa_co2 using mcp;

*       policy solves with carbon taxes at $50/ton increments

        carbtax             =       50 * co2_carb * (ord(iter) - 1);
        carblim$carbtax     =       1;

$include usa_co2.gen
        solve usa_co2 using mcp;

        results(iter,"pcarb")       =       pcarb.l / co2_carb;
        results(iter,"emissions")   =       1e4 *(carblim * ctax.l +
                                                    carblim0$(carbtax = 0));
        results(iter,"abatement")   =       1e4 * carblim0 -
                                                    results(iter,"emissions");
        results(iter,"cons")=       ra.l / 100;
        results(iter,"% ev")=       (ra.l / ra0 - 1) * 100;
        results(iter,"gdp") =       (ra.l + sum(id, sum(ne, p.l(ne) *
                                            g0(ne,id)) + sum(e, pce.l(e) * g0(e,id)))
                                            + sum(i, p.l(i) * nx0(i))) / 100;
        results(iter,"% gdp")       =       ((ra.l + sum(id, sum(ne, p.l(ne) *
                                            g0(ne,id)) + sum(e, pce.l(e) * g0(e,id))) +
                                            sum(i, p.l(i) * nx0(i))) /
                                            (ra.l + sum(id, sum(ne, g0(ne,id)) +
                                            sum(e, g0(e,id))) + sum(i, nx0(i))) - 1) *
                                            100;

        p_impacts(iter,"pcarb")     =       results(iter,"pcarb");
        p_impacts(iter,ne)  =       (p.l(ne) - 1) * 100;
        p_impacts(iter,e)           =       (pce.l(e) - 1) * 100;

        q_impacts(iter,"pcarb")     =       results(iter,"pcarb");
        q_impacts(iter,i)           =       (qd.l(i) / y0(i) - 1) * 100;

        c_impacts(iter,"pcarb")     =       results(iter,"pcarb");
        c_impacts(iter,ne)  =       (necons.l(ne) / sum(cd,g0(ne,cd)) - 1) *
                                            100;
        c_impacts(iter,e)           =       (econs.l(e) / sum(cd,g0(e,cd)) - 1) * 100;

        coal_impacts(iter,"pcarb")        =       results(iter,"pcarb");
        coal_impacts(iter,i)$x0("col",i)  =       (qeint.l("col",i) /
                                                          x0("col",i) - 1) * 100;

        oil_impacts(iter,"pcarb")  =       results(iter,"pcarb");
        oil_impacts(iter,i)$x0("oil",i)  =       (qeint.l("oil",i) /
                                                          x0("oil",i) - 1) * 100;

        gas_impacts(iter,"pcarb")  =       results(iter,"pcarb");
        gas_impacts(iter,i)$x0("gas",i)  =       (qeint.l("gas",i) /
                                                          x0("gas",i) - 1) * 100;

        elec_impacts(iter,"pcarb") =       results(iter,"pcarb");
        elec_impacts(iter,i)$x0("ele",i) =       (qeint.l("ele",i) /
                                                          x0("ele",i) - 1) * 100;
```

```
      emiss(iter,"pcarb") =     results(iter,"pcarb");
      emiss(iter,i)       =     1e4 * sum(e,ccoef(e) * qeint.l(e,i));
      emiss(iter,"hhold") =     1e4 * sum(e,ccoef(e) * econs.l(e));

      mac(iter,"pcarb")         =     results(iter,"pcarb");
      mac(iter,i)               =     1e4 * sum(e,ccoef(e) * (x0(e,i) -
                                      qeint.l(e,i)));
      mac(iter,"hhold")         =     1e4 * sum(e,ccoef(e) * (sum(cd,g0(e,cd)) -
                                      econs.l(e)));
);

file usa_co2_results;
put usa_co2_results;
usa_co2_results.pc  =     6;
usa_co2_results.pw  =     3000;

$libinclude gams2tbl results
$libinclude gams2tbl p_impacts
$libinclude gams2tbl q_impacts
$libinclude gams2tbl c_impacts
$libinclude gams2tbl coal_impacts
$libinclude gams2tbl oil_impacts
$libinclude gams2tbl gas_impacts
$libinclude gams2tbl elec_impacts
$libinclude gams2tbl emiss
$libinclude gams2tbl mac

putclose usa_co2_results;
```